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THE PREPARATION OF COPPERED GLASS MIRRORS

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[Communicated by Major C. H. S. EVANS, O.B.E., R.E. (T.A.).]

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ABSTRACT. The object of this paper is to introduce a method for the deposition of copper on plate glass surfaces, in the form of a highly reflecting coherent film, with special reference to searchlight mirrors.

An earlier research on this subject was conducted by Chattaway in 1907, using phenylhydrazine as a reducing agent, but on his findings it was not possible to obtain a coherent copper film on plate glass surfaces. It is considered by the author of this paper that this was probably due to the formation of tarry by-products, which appear inevitably to result when such a reducing agent is used. A considerable amount of research was conducted at the War Department Searchlight Experimental Establishment with the object of eliminating this drawback, as well as temperature difficulties, insuperable from the use of phenylhydrazine. During this research all possible variations of temperature reduction, quantity and quality of chemicals were explored.

This paper presents in detail the results of a few of the principal experiments which were conducted during the necessary research, and fully defines a new method which, by the use of hydrazine sulphate, enables a satisfactory coherent and lasting film to be deposited on plate glass surfaces, at a comparatively low temperature.

As it appears that the cleaning of the glass surface is peculiarly important to the film deposited thereon, a short statement is made as to the cleaning method which was found to be conducive to a satisfactory copper film.

Introduction

THE deposition of copper in a brightly reflecting coherent form from the solutions of its salts offers difficulties not encountered in the case of silver. Copper films were first obtained by Faraday, who in 1857 produced mirrors by heating plates of glass in a mixture composed of oxide of copper and olive oil, deposition occurring when the oil decomposed. The mirrors obtained in this way were, however, very poor, as they lacked evenness and brilliance. Faraday* also discovered that a copper film could be produced by "spattering" the metal on to the glass, in an atmosphere of hydrogen, by means of a battery of Leyden jars.

In 1895 Walther† drew attention to the application of phenylhydrazine as a reducing agent in organic chemistry, and Chattaway‡ stated that "hydrazines of the formula R. N. H. NH_2, where R is a cyclic group, are readily oxidised by free oxygen, the action being accelerated in a remarkable way by alkali hydroxide. In the case of phenylhydrazine dissolved in three times its weight of 4N alcoholic potash, oxidation is so rapid that the liquid effervesces freely from escape of nitrogen on mere exposure to air." Again he states§ that primary aromatic hydrazines are readily attacked by most substances capable of giving up oxygen at all easily, the products being in the main the same as those which are formed by free oxygen.

The main reaction, which results in the replacement of the hydrazine group by hydrogen and the simultaneous liberation of nitrogen, is generally accompanied by secondary actions, which occur, however, to a relatively very small extent.

It appeared, therefore, that phenylhydrazine offered some promise of successful employment especially since its oxidation may be greatly accelerated by the presence of caustic alkali. In 1907 Chattaway|| described a method of obtaining copper mirrors from aqueous solutions of copper salts, using phenylhydrazine as a reducing agent in conjunction with potassium hydroxide. The process was found, however, to be successfully applicable only to the inner surfaces of blown glass vessels. It appeared to be impossible to obtain satisfactory deposits on plane surfaces, e.g. plate glass, owing to interference by the tarry by-products which were formed. These results, although limited, appear to confirm the promising nature of phenylhydrazine as a reducing agent, and the following work was carried out to find a satisfactory method for the production of coppered glass reflectors.

Preliminary Experiments

As an example of the method pursued, the following may be quoted:

A solution was prepared by mixing boiling distilled water with phenylhydrazine, which, it was found, had to be pure, for if impure phenylhydrazine was used the resulting solution was of a dirty yellowish colour. This was overcome by using phenylhydrazine freshly distilled under diminished pressure, with the result that a perfectly clear solution was obtained. It was found, however, that certain samples of phenylhydrazine, although freshly distilled, still presented a slight turbidity when mixed with water.

Investigations into the cause of this turbidity then showed that it was due to what is called the "oil" of phenylhydrazine becoming mixed with the solution. This oil floats to the surface of the phenylhydrazine after the latter has frozen, i.e. below 21° C.

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* *Phil. Trans.* 147 (1857) 154.

† *Journ. Prakt. Chem.* (1895) 141.

‡ *Proc. Chem. Soc.* 23 (1907) 183.

§ *Proc. Chem. Soc.* 24 (1908) 10.

|| *Proc. Roy. Soc.* 80 (1907–8) 88.

A fresh sample of the same phenylhydrazine was then taken in the frozen condition and the oil on the surface poured off. The solid phenylhydrazine which then remained was melted and a sample of it mixed with boiling water, with the result that a perfectly clear solution was obtained. These tests therefore proved that, to obtain a clear solution of phenylhydrazine and water, the former must be freshly distilled and freed from this oil by crystallization.

It was found that this solution was better obtained by heating phenylhydrazine and water together, until the turbidity, first produced when cold, disappeared, leaving a perfectly clear solution. This occurred when the mixture was at a temperature of about 70° to 80° C.

A 10 per cent. solution (by weight) of potassium hydroxide was then prepared, using stick caustic (pure by alcohol). Sodium hydroxide was tried, but was found to give inferior results.

The third solution required consisted of ·880 ammonia solution to which had been added sufficient copper hydroxide to saturate it, a slight excess of the copper hydroxide being added to make certain of complete saturation. As cuprammonium solution cannot be filtered through paper—or indeed through any cellulosic medium—the excess of copper hydroxide in the latter solution was separated, either by filtration through closely packed glass wool or by decantation.

The hot solution of phenylhydrazine in water was mixed with the saturated ammoniacal solution of copper hydroxide. To this was added the 10 per cent. caustic potash solution until slight precipitation was produced. The surfaces to be coppered were placed in the mixture, which was kept just under its boiling point.

Effect of Temperature and Composition of Solutions

Many experiments were made in which such factors as were likely to affect the results were explored, e.g. temperature and composition of the liquids. The best results appeared to be obtained when the following proportions and conditions were realized.

Phenylhydrazine	1	part by volume
Hot 10 per cent. solution (by weight) potassium hydroxide	0·585	,,
Solution of copper hydroxide and ·880 ammonia	2	parts by volume
Water, distilled	5	,,

The phenylhydrazine, after having been mixed with the water, was heated until the mixture became a quite clear solution. To this was added the solution of copper hydroxide in ammonia, and then the 10 per cent. solution of potassium hydroxide, when a reddish-yellow turbidity or precipitate was formed. This solution was then heated to boiling point in contact with the plate glass surface to be coppered. The deposit obtained, however, was very thin, uneven, and tarnished, due entirely to the effect on the film of the tarry by-products which were formed.

It was found, however, that these proportions had to be strictly abided by to obtain a deposit at all on plate glass, since it appears from experiments that there is a considerable difference between depositing copper on the surface of plate glass and on blown glass. This difference occurs in the case of silver to some extent, but is considerably more marked in that of copper.

Choice of Reducing Agent

The great difficulties experienced in the use of phenylhydrazine as a reducing agent were:

(1) The formation of tarry by-products, which were produced in every case, and clouded the mirror.

(2) The high temperature which, in every experiment, was found necessary for the reduction of the copper salt.

(3) During the process very irritating fumes were evolved and, on account of the bad effect which the solution has on the skin, great care had to be taken to protect the operator's hands.

Though numerous experiments were made, none of the results could be regarded as satisfactory. This is in conformity with Chattaway's experience, and indicated that other methods were necessary to obtain the desired results. It appeared from these results that phenylhydrazine as a reducing agent did not lead to the production of satisfactory copper films on plate glass, and a new reducing agent was therefore sought.

It appeared of interest to investigate the parent substance, and so hydrazine sulphate was used as a suitable compound which, compared with phenylhydrazine, has greater solubility in water, is cheaper, less poisonous, and is more easily purified.

A 10 per cent. solution of hydrazine sulphate was made, to which was added an equal volume of a saturated solution of copper hydroxide in ·880 ammonia, prepared as previously explained. The resulting solution, which was of a perfectly clear yellow colour, was divided into six test tubes, and a 10 per cent. solution of potassium hydroxide (prepared by weight from sticks, pure by alcohol) at a temperature of 60° C. was added to each tube, the amount added varying from $\frac{1}{2}$ to 3 c.c., increasing by $\frac{1}{2}$ c.c. in each case. The addition of the latter produced a turbidity in every tube.

The six tubes were then heated by immersion in a beaker of boiling water. After a few minutes the solution in tubes 5 and 6, i.e. the two which contained the most potassium hydroxide, changed colour from turbid yellow to a dark greenish colour, and almost directly at this point copper commenced to deposit on the sides of the two test tubes. The other four tubes, however, did not show any signs of change, although much later there was a change of colour of the solutions they contained.

The result of this experiment showed that, by increasing the amount of potassium hydroxide, the copper was not only deposited, but the deposition was relatively accelerated. There was, moreover, not the slightest suspicion of tar or scum.

This experiment was repeated, using the same copper-hydrazine solution, but adding more alkali, in this case 3 to 8 c.c. being employed. The test tubes were heated in boiling water, and the same action as before took place. Tubes 4, 5, and 6 all presented a fairly good deposit of copper, whilst tubes 1, 2, and 3 were inferior.

A mixture was then made up to the quantities of tube 6, that is:

(1) Water distilled, 10 c.c., hydrazine sulphate, 1 gm.
(2) 10 per cent. solution of potassium hydroxide, at 60° C., 8 c.c.
(3) Saturated solution of copper hydroxide in ·880 ammonia, 10 c.c.

The hydrazine sulphate solution was heated to 60° C., and the saturated solution of copper hydroxide in ·880 ammonia added to it. The potassium hydroxide solution was then added to this mixture. The resulting solution, which was turbid and yellow, was poured upon a piece of plate glass in a copper vessel, which was capable of being heated in a water bath, as shown in Fig. 1. The water bath was provided with a thermometer and was heated to 82° C.; only a small amount of copper was deposited on the glass.

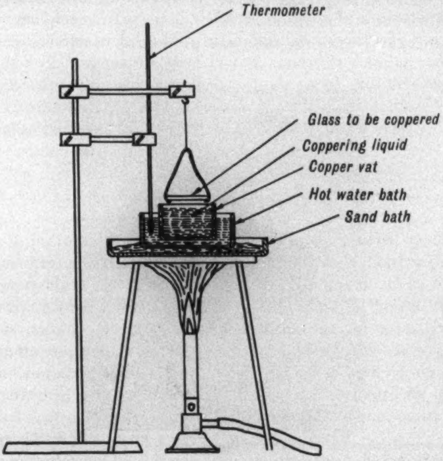


Fig. 1

Other tests were then carried out under exactly the same conditions, with the exception that the temperature of the water bath (i.e. the temperature of the mixture) was varied, when it was found that at a temperature between 43° and 57° C. the result of the deposition was superior, although the copper film was not of sufficient thickness.

It was found, however, that success depended upon the temperature of the solution and the amount of potassium hydroxide solution added being slightly less than that required just to produce the turbidity or precipitate which occurs when it is added in excess to the mixed solutions of hydrazine sulphate and copper hydroxide in ammonia, i.e. just before the precipitate is formed.

The same process as just explained was repeated, only in this case a saturated solution of copper tartrate (made fully alkaline by the addition of a few drops of ammonia) was used. The result of depositing copper on ordinary plate glass was, however, not good, since the film in this case tended to blister, and so the tartrate solution of copper was abandoned.

After several further tests with the original reagent, it was found that the reduction of the mixed solution did not occur at a definite temperature, but between the limits of 43° and 57° C., for lower than 43° very little reduction takes place, whilst higher than 57° the copper is not deposited on the glass surface as a reflecting film, but is thrown down as a fine granular precipitate. This was of great importance, and led to the successful production of mirrors.

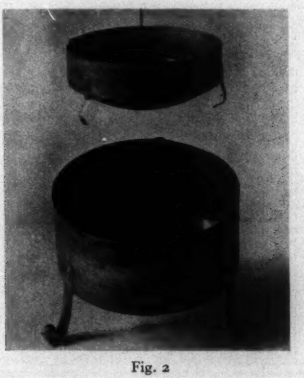


Fig. 2

Method of Coppering

The mirror to be coppered is first fixed to a float, as in Fig. 2, and the vat—provided with legs as shown—which is capable of receiving the float with an annular space all round, is then placed in a water bath capable of being heated to the required temperature. The following solution is then made:

9 gm. of hydrazine sulphate are dissolved in 150 c.c. of distilled water, and heated to a temperature of 60° C.; to this is added, with constant stiring, 90 c.c. of the saturated solution of copper hydroxide and ammonia at 16° C. The resulting mixture will present a clear yellow colour.

The glass, after having been treated by the cleaning process explained later, is then sponged over with the mixture. This sponging is very important, for it has been found that it is necessary to give satisfactory results.

Without delay, to the mixture of the two solutions is added slowly, and with constant stirring, 87·5 c.c. of a solution (at 60° C.) consisting of 11·1 gm. of potassium hydroxide (pure by alcohol) to every 100 c.c. of distilled water. If this is correctly carried out, the final mixture of the solutions should still be of a clear yellow colour, without any signs of precipitation.

The mixture is then poured into the copper vat (Fig. 2), which must be standing in a bath of hot water (not shown) at a temperature of 43° C. The mirror to be coppered, which is clipped to the float, is then floated and periodically given a rotary motion in the liquid. Great care must be taken to see that very little time elapses between the time of adding the potassium hydroxide solution and that of floating the mirror to be coppered. The temperature of the water bath (i.e. the bath which heats the coppering solution) is then very gradually raised to 57° C. During this rise of temperature, and in fact almost as soon as the glass is in, the liquid in the copper vat will commence to turn a dark green colour and remain so for about 15 minutes, after which its colour commences to turn a pink shade, the whole change taking about 20 minutes, by which time a copper film will have formed on the surface of the glass.

Thickening the Film

In order to protect the film from damage, and to prevent oxidation by the atmosphere, thickening is necessary. For this purpose the following process, which also adds appreciably to the reflective power of the film, should be followed:

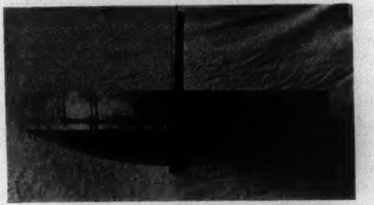
The mirror, which is already floating in the solution in the vat, should be left for about five minutes, when the temperature of the vat must be dropped to 43° C., and to the remaining solution in the vat an addition made of 50 c.c. copper hydroxide and ammonia solution at 16° C. to which has been added 10 c.c. of the potassium hydroxide solution at 60° C. This mixture is added by quickly lifting the mirror and float out of the vat and, with constant stirring, pouring it into the solution, after which the mirror must be quickly replaced.

After the mirror is put back it will be found that the solution in the vat, which should be approximately at 43° C., will again turn black and, if gradually heated to 50° C., further reduction of copper will take place, which will slightly thicken the previous film.

By exactly the same process as that just described, the temperature is dropped to 43° C. and another addition is made by pouring into the vat a solution consisting of 1 gm. of hydrazine sulphate dissolved in 60 c.c. of copper hydroxide and ammonia solution at 16° C. Again after the temperature is raised to 50° C. the colour of the resulting mixture changes. After 15 minutes the temperature is again dropped to 43° C. and another addition is made similar to the first, i.e. 50 c.c. copper hydroxide and ammonia solution at 16° C., to which has been added 10 c.c. potassium hydroxide solution at 60° C.

A period of 15 minutes is then allowed to elapse, during which the temperature is gently raised to 50° C.

Again, after the temperature of the solution in the vat has dropped to 43° C., another addition is made similar to that of the second, that is, a solution is added consisting of 1 gm. hydrazine sulphate dissolved in 60 c.c. of copper hydroxide in ammonia solution at 16° C., and after again heating the vat to 50° C., the whole is allowed to cool to the temperature of the room before the mirror is finally taken out.



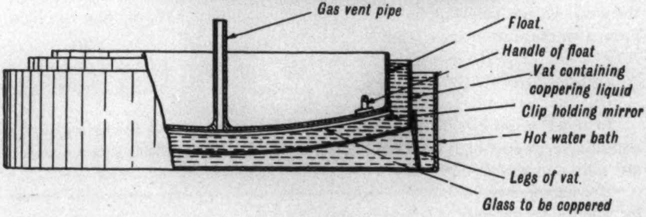


Fig. 3

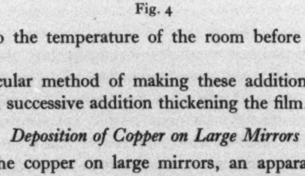


Fig. 4

The above particular method of making these additions was found to give the best results, each successive addition thickening the film.

Deposition of Copper on Large Mirrors

For depositing the copper on large mirrors, an apparatus may be used as shown in Fig. 3. It consists of a vat for holding the coppering liquid, and a float upon which the mirror is mounted. The mirror is fixed to the float by 24 clips, which are provided with slots through which a copper wire can pass, as shown in Fig. 4. This wire is made fast so as to draw all the clips up to the mirror, and also to prevent them bending outwards. The float is also provided with a pair of handles and a central tube through which any gas that is generated in the vat can escape.

The vat is of sufficient size to provide an annular space of about 1½ inches, to allow any bubbles or solid matter that may form to float to the surface of the liquid, and thus not come into contact with the surface of the glass upon which copper is to be deposited. It is constructed to stand in a water bath capable of being heated up to the desired temperature, and allowing the water to circulate underneath and around the sides.

Thickening by Electro-deposition

As a final protection to the films, the coppered mirror, after having been washed in ordinary and distilled water, is dried and placed in an electroplating bath, and copper electrically deposited for half-an-hour. Under no circumstances must this electro-coppering be done in an ordinary cyanide or acid copper sulphate bath, as such baths cause the copper film to strip. It is therefore essential that a bath should be used free from any marked tendency to cause solution of the film. A satisfactory one for this purpose is that of Dr Elsoner, which is as follows:

One part by weight of bitartrate of potassium is boiled in ten parts by weight of water. As much hydrated carbonate of copper is then added as the liquid will dissolve. This is then made slightly alkaline by the addition of a small quantity of carbonate of potassium.

Great care must be taken not to allow the current density at the cathode to become too great, for, if it does, there is a danger of stripping the film. A safe current density to use is 0·25 ampere per square decimetre.

In the case of parabolic mirrors, coppering is best done by putting the mirror back on the float and floating the whole in the copper vat which, after being cleaned, should be partially filled with the electrolyte.

The whole deposit can be finally protected by lead backing if desired.

Preparation of Glass Surface (upon which copper is to be deposited)

The glass surface is first thoroughly cleansed with concentrated nitric acid, then, after having been thoroughly washed with tap water, it is placed for half-an-hour in a warm 1 per cent. solution of caustic potash; it is then taken out, washed well with ordinary water for about half-an-hour, and finally rinsed with distilled water. The whole surface is then thoroughly polished with a perfectly clean, dry glass cloth, which should have been previously washed in the following manner:

The cloth is first boiled in water with castille soap, after which it is removed with metal tongs and boiled with successive changes of water until no trace of milkiness is perceptible. This method, although laborious, is very efficient for removing grease, and the washed cloth after drying must be kept in a grease-free bottle, handled with tongs alone, and used once only before rewashing.

SUMMARY AND CONCLUSIONS

(1) A method has been described by which plate glass may be coppered.

(2) Hydrazine sulphate is superior to phenylhydrazine as a reducing agent for this purpose.

(3) The limits of temperature within which highly reflecting deposits are obtainable have been indicated.

(4) The process now described is appreciably cheaper than that in which phenylhydrazine is employed.

(5) The use of hydrazine sulphate prevents the formation of scum and stains, and is much safer for the operator.

I desire to thank the President, Royal Engineer Board, for permission to publish these results. I wish also to record my appreciation of the facilities for the necessary experimental work kindly granted by Major C. H. S. Evans, O.B.E., R.E. (T.A.), Superintendent, Searchlight Experimental Establishment, and the assistance received from Lieut. H. C. Gibson, R.E. (T.A.) and Mr S. Field, A.R.C.S., in other ways.

DISCUSSION

Major C. H. S. Evans: The Searchlight Experimental Establishment has for several years experimented with various reflecting surfaces, and Mr French's research described in this paper was the outcome of a demand for a surface capable of producing, to a certain extent at least, selective reflection.

Though Mr French pursued his research to a very satisfactory practical conclusion, other factors precluded the adoption of coppered glass mirrors for searchlight work, e.g., the considerable heat absorption, endangering the glass, and the low reflective efficiency, which latter was of the order of 65 per cent. compared with 90 per cent. in the case of a silvered glass mirror.

There seems a practical commercial utility in the process for motor-car headlight reflectors, and there is no doubt that the copper reflecting surface is of a very pleasing warm colour for use as an ordinary household decorative mirror.

I would add to Mr French's emphasis on scrupulous cleanliness throughout the process; my own confirmed opinion is that this is essential to success, as in the majority of similar processes involving the deposition of reflecting surfaces on glass; it cannot be too strongly stressed. In the case of searchlight mirrors, the temperature reached by glass and reflecting surfaces is often as high as 250° C. in practice, and the effects of the slightest lack of cleanliness in the deposition process, and non-observance of the other points of procedure described in the paper, rapidly show up in accelerated disintegration of the reflecting surface itself, and lowered reflective efficiency after use, either of the whole, or of patches of the mirror surfaces.

Capt. S. P. Holloway: Will Mr French please indicate the difference in the final result due to the swabbing, as some workers place the glass in the depositing fluid in a wet state after swilling with distilled water? Does not this method also break down the surface tension?

Mr F. F. S. Bryson: What part does the alkali take in the mechanism of the process, and has it any dissolving action on the surface of the glass, thus rendering the coppering more effective? Glass may be impregnated by copper as a result of electrolysis. If the glass be heated to about 300° C. and a current passed from a copper amalgam as anode on one side of the glass to a mercury cathode on the other, the copper from the amalgam enters the glass to take the place of the sodium removed by electrolysis. This method, of course, could not be used for the preparation of mirrors since the resulting surface is usually dull and patchy and the introduction of the copper tends to make the glass brittle.

Mr French: Major Evans' contribution to the discussion is complementary and explanatory and does not need any further reference by me, except that the reflective efficiency of 65 per cent. mentioned by him is the figure arrived at as measured through the glass, which absorbs about 10 per cent. of the light.

In reply to Capt. Holloway, it was found by experiment that the process of drying and polishing the glass surface with a clean glass cloth and afterwards breaking down the surface tension by swabbing materially improved the copper film. If the glass was only swilled with distilled water, then, although the surface tension was broken down, the copper film was not so good as that obtained by the process detailed in this paper.

In reply to Mr Bryson, the author is of the opinion, as a result of continued investigation, that the caustic alkali acts as an accelerator to the reducing fluid and does not have any appreciable dissolving action on the glass. If an excess of potassium hydroxide is added, the final mixture precipitates cuprous hydroxide with the result that the copper film is poor. It is for this reason that the exact quantity and quality of potassium hydroxide are stated in this paper.

THE TAYLOR-HOBSON *F*/2 ANASTIGMAT

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MS. received, 9*th April*, 1924. *Read and discussed*, 8*th May*, 1924.

ABSTRACT. An analysis of the aberrations of an anastigmat lens having an aperture of *F*/2 is given, and comparison made with existing large aperture lenses. Some examples of the performance of the actual lens as manufactured are added, and directions in which the lens should be useful are suggested.

Introduction

THE need for greater sensitivity of the camera in recording objects in motion is apparent enough when we consider the number of scenes which are visible to the eye but to the camera are practically invisible. The speeding-up of the camera depends on the optician and the plate maker, and their labours can never be considered at an end till everything which the eye can see can be recorded on the photographic plate. This means that maximum sensitivity of the plate must be made to fall at the wave-length for which the eye is most sensitive. Attainment of this ideal is at present only approximate and at the expense, to some extent at least, of speed. When such plates are in general use the optician will have to modify the corrections of his lenses; at present his task is complicated by having to adopt a compromise between visual and photographic correction. The usual course for the lens designer is to calculate the performance of his system for the hydrogen *G*′ line and check the result by a photographic test.

Specification of Lens

The aperture *F*/2 is not in itself remarkably great, but a photographic lens must embrace a field of at least 35°. Actually the Cooke lens covers over 50° with a flat field at this aperture. In deciding to aim at a large aperture, axial corrections are the first consideration, and we have to choose a type of lens which possesses small zonal spherical aberration and small chromatic difference of spherical aberration. The Gauss type was considered most favourable and its symmetrical shape makes its design less laborious. The half system was first designed to be approximately correct in itself, and then the symmetrical system was afterwards modified to give the best corrections for the lens as a whole. This led to a somewhat large departure from exact symmetry as the specification* for a lens of unit focal length will show:

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* *Brit. Pat.* 157,040 (1920).

Radii	Thicknesses and separations	n_D	ν
0·65335			
	0·0836	1·6135	59·5
3·0243			
	0·0139	1	
0·44614			
	0·0975	1·6135	59·5
∞			
	0·0418	1·576	41·2
0·2869			
	0·1393	1	
−0·2869			
	0·0418	1·576	41·2
∞			
	0·0975	1·6135	59·5
−0·38786			
	0·0139	1	
∞			
	0·0836	1·6135	59·5
−0·68839			

Fig. 1 shows the lens in section.

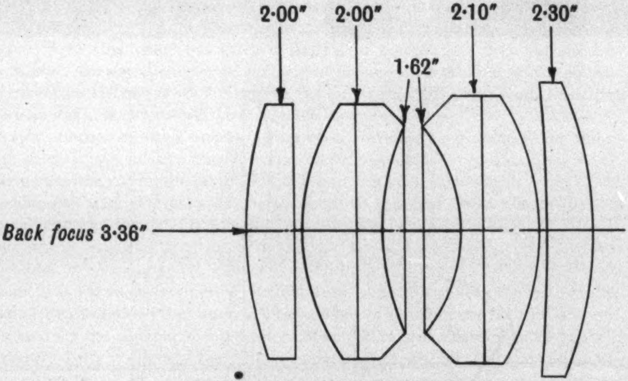

Fig. 1*. 4½" Ser. O. *F*/2. Scale full-size

One often sees reference to lack of depth of field in large aperture lenses, and some rapid lenses claim specially great depth. Usually discussion of the question tacitly presupposes perfect image formation, which of course does not exist in practice. The effect of aberration of the image on the depth of field is a question hardly worth consideration. It is sometimes overlooked that depth depends on the diameter of the lens in use and not on its relative aperture, so that a lens of half the focus with double the relative aperture retains the advantage of its superior speed without the disadvantage of lack of depth. The only difference is in the size of image. If the smaller focus lens yields sufficiently good definition, the smaller image can afterwards be enlarged. As an example are shown two photographs (Fig. 2) of a series of objects placed at different distances from the lens, taken, one with a 7 inch lens at *F*/4·5, the other with a 3 inch lens at *F*/2, the one image being enlarged to the same size as the other. Very little difference in depth of focus can be seen between the two.

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* There is a slight error in the figure; the two central surfaces should have the same radius of curvature.

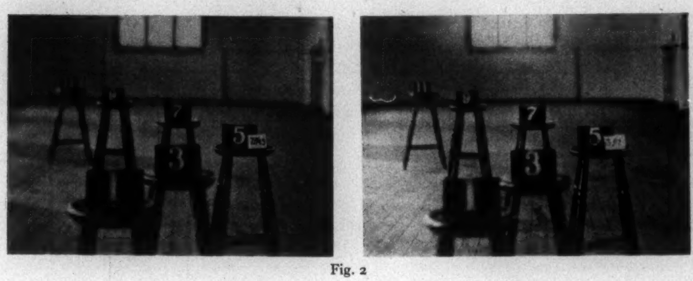
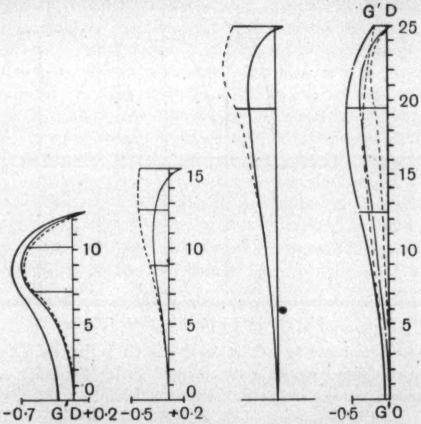


Fig. 2



94 *a*. 146 *a*. Cooke Ser. O.

Fig. 3

Spherical Aberration

The curves for spherical aberration (full curve) and departure from fulfilment of the sine condition (dotted curve) are given in Fig. 3 on the same scale as similar curves in von Rohr's *Theory and History of the Photographic Lens*. Two sets of curves from that book are reproduced by the side of those for the Cooke lens for comparison. They are those of two large aperture anastigmats, the *F*/3·7 of H. D. Taylor (94 *a*) and the *F*/3·3 Planar of Rudolph (146 *a*). It may be well to be cautious in accepting the figures taken from a Patent Specification, as the data there disclosed may not refer to the best adjustment of the lens, but a general idea of its performance may be gathered, especially as to the run of the higher aberrations. It will be noticed that the Cooke *F*/2 lens displays the same form of longitudinal aberrations as the Planar and that there is an increasing departure from exact fulfilment of the sine condition with increasing aperture. This has been found to give the best correction for coma. It is frequently found that the satisfaction of the sine condition in large aperture lenses is no very exact criterion of freedom from coma. The illustration also shows how the curves would run if the sine condition were satisfied at full aperture with the corresponding curves for the hydrogen *G′* line. It will be seen that the *G′* curve follows that for *D* very closely, which is a good indication that the correction for still shorter waves will be good. It was proved, in fact, by a photographic test that the best general focus for the rays emitted from an enclosed arc lamp coincide with the best focus for the visible spectrum. Exactly where that occurs I do not know. S. D. Chalmers[*] has given a method of theoretically deducing the best focus from the lateral aberration curves, but no account was taken of chromatism, which must play a large part in helping the eye to decide the best focus. Research work in this direction would be invaluable.

For comparison the *G′* spherical aberration curve has been added to von Rohr's Fig. 94 *a*. It will be seen that, to counteract the spherical over-correction for *G′*, a large amount of under-correction for colour has been introduced.

The algebraic expression of the longitudinal spherical aberration of the new lens as deduced from the results of trigonometrical computation is

$$Y = -\tfrac{1}{2} f A^2 ({\cdot}419 - 16{\cdot}43 A^2 + 467{\cdot}8 A^4 - 5115 A^6),$$

where A is the semi-aperture. (The first term is taken from an algebraic evaluation of the first order aberration.) The number of terms in the expansion is conditioned by the number of rays traced, but the various terms may be taken to represent fairly accurately the different orders of aberration. The corresponding expression for *G′* is

$$Y = -\tfrac{1}{2} f A^2 ({\cdot}444 - 1{\cdot}945 A^2 - 99{\cdot}28 A^4 + 412 A^6).$$

The following table exhibits what part the various orders contribute to the total:

Aperture		1st order	2nd order	3rd order	4th order	Calculated aberration
F/2	*D*	−·01309	+·03209	−·05709	+·03903	+·00090
	G′	−·01387	+·00381	+·01213	−·00014	−·00199
F/2·64	*D*	−·00753	+·01053	−·01071	+·00423	−·00348
	G′	−·00796	+·00125	+·00229	−·00034	−·00477
F/4	*D*	−·00327	+·00201	−·00089	+·00015	−·00201
	G′	−·00347	+·00024	+·00019	−·00001	−·00305

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[*] *Trans. Opt. Soc.* 18 (1917) 189 *et seq.*

Sine Condition

Departure from the fulfilment of the sine condition is exhibited by the divergence of the dotted curve from the spherical aberration curve. The amount is found to be expressed algebraically by the equations:

$$y = -\tfrac{1}{2} f A^2 (\cdot 013 - 1\cdot 836 A^2 + 152\cdot 2 A^4 - 1171 A^6) \text{ for } D,$$

and
$$y = -\tfrac{1}{2} f A^2 (-\cdot 037 + 4\cdot 461 A^2 - 75\cdot 72 A^4 + 991\cdot 2 A^6) \text{ for } G'$$

and the contributions of the various orders are:

Aperture		1st order	2nd order	3rd order	4th order	Total
F/2	*D*	− ·00041	+ ·00358	− ·01850	+ ·00893	− ·00646
	G′	+ ·00116	− ·00871	+ ·00924	− ·00756	− ·00588
F/2·64	*D*	− ·00023	+ ·00118	− ·00351	+ ·00097	− ·00160
	G′	+ ·00066	− ·00287	+ ·00175	− ·00082	− ·00130
F/4	*D*	− ·00011	+ ·00022	− ·00029	+ ·00003	− ·00016
	G′	+ ·00029	− ·00054	+ ·00015	− ·00003	− ·00014

It will be noticed that the higher aberrations for *G′* are not even of the same sign as the corresponding ones for *D*, and further that the algebraic series does not rapidly converge. This last fact seems to point to the hopelessness of trying to design a lens of this aperture and field from purely algebraic equations. The first order aberrations must be approximately reduced to zero, and this forms a first and most important step in the design; the rest is done by trigonometry and modification as the result of test of a model.

Coma

Coma is checked by tracing three initially parallel rays through the system at a convenient angle. The result of such a calculation is given for oblique rays at 17° to the axis. h_1, h_2, and h_3 are the intercepts on the focal plane of the rays passing through the top, middle, and bottom of the effective aperture respectively.

$$\begin{aligned} h_1 &= \cdot 30750 & \tfrac{1}{2}(h_1 + h_3) &= \cdot 30416 \\ h_3 &= \underline{\cdot 30082} & h_2 &= \underline{\cdot 30717} \\ \text{Spherical aberration} &= \underline{\cdot 00668} & \text{Coma} &= \underline{\cdot 00301} \end{aligned}$$

Aperture of beam is ·26017 = *F*/3·8.

From the axial calculation we obtain longitudinal spherical aberration	− ·00217
Focus for a thin pencil, i.e. *M* aberration at 17°·3, is (see next page)...	− ·00044
Total longitudinal aberration is	− ·00256
giving a lateral under-corrected aberration of	·00066
Over-correction of spherical aberration of oblique pencil is	·00734

Similarly for coma the axial expression gives us the inward coma at this aperture

	·00016
From the above it is	·00301
so that	·00285

is the inward coma due to the oblique pencil.

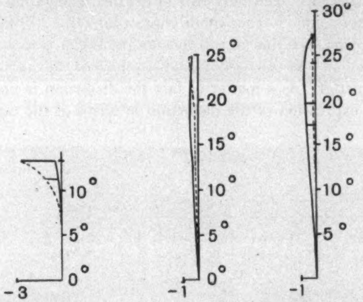
The corresponding values for G' are

$$h_1 = \cdot 30707, \quad h_2 = \cdot 30677, \quad h_3 = \cdot 30076.$$

Again it is seen that the G' rays keep close to the D rays. The value of the coma deduced from the axial expression is a very small part of that actually found; in fact the higher order oblique aberrations are preponderant.

Field

Curves are given in Fig. 4 showing the field, again on the same scale as used by von Rohr, and with reproductions of his curves for the same two lenses for comparison. The resemblance between the Cooke and the Planar is again apparent.



94 *b*. 146 *b*. Cooke Ser. O.

Fig. 4

The aberrations for the meridional (dotted) and sagittal (full line) rays can be represented as far as 25° from the axis by the equations

$$m = - \cdot 0112 t^2 + \cdot 09 t^4 - \cdot 25 t^6,$$
$$s = - \cdot 1016 t^2 + \cdot 39 t^4 - \cdot 083 t^6,$$

where the first term is taken from algebraic calculation of the first order aberration coefficients and t is the tangent of the semi-angle of field. Beyond 25° this expression no longer represents the aberration. The contributions of the various terms are shown in the annexed table:

Angle on Object side	1st order		2nd order		3rd order		Aberration calculated	
	M	*S*	*M*	*S*	*M*	*S*	*M*	*S*
17°·3	−·00105	−·00956	+·00078	+·00345	−·00021	−·00007	−·00044	−·00624
19°·7	143	1304	146	643	53	16	50	673
24°·1	224	2043	357	1578	203	68	69	519
27°·1	293	2666	608	2087	450	150	220	005

Again it is found that the field for G' is very similar, e.g. the aberrations for the first angle are respectively − ·00597 and −·00055.

Where higher order axial aberrations play so large a part, too much importance must not be paid to these figures, which presuppose a small stop. It is likely from the calculation with three rays that the full pencil focusses somewhat flatter than the small pencil.

Distortion

Coma and astigmatism depending on aperture and field of view, it is not possible to form a very exact impression of the performance of the lens from trigonometrical computation, but in dealing with distortion we have only to consider field. The following table shows the distortion and it will not be very surprising to find that we need only two terms in the algebraic expansion. They are both of the same sign, so that the higher order terms do not compensate the lower. It may be remarked that in this lens no particular care was made to correct for distortion, which for the purposes for which the lens was likely to be used would not be a serious matter. As a matter of fact the distortion is nowhere large.

The algebraic expression of the distortion in terms of the tangent of the angle of field *t* is

$$x = \cdot 0107t^3 + \cdot 057t^5.$$

Angle	1st order	2nd order	Calculated distortion	Distortion for *G′*
17°·3	+·00031	+·00015	+·00045	+·00064
19°·7	49	34	83	
24°·1	97	103	202	
27°·1	144	201	364	

General Definition

The ultimate test of a photographic lens is the quality of the photograph that can be made with it. A test photograph made with this lens shows sharp definition over the whole field. One was shown on the screen at the meeting, but is not reproduced here, as a half-tone block is not suitable for showing fine definition.

Possible Uses of Lens

In conclusion two pictures are shown (Figs. 5 and 6) which illustrate one direction in which the lens has found a commercial use, i.e. in theatrical photography. It is possible now to photograph the actual performance on the stage with the ordinary stage lighting. These photographs have more historical and histrionic value than reconstructed flashlight photographs.

In kinematography the lens should be valuable, especially in the studio, permitting of photography with less intensely actinic light, which has been found so harmful to the actors' eyes. Fresh ground is being broken in other directions, though developments are not so far advanced as to permit of further reference at present.

In astronomy the new lens should prove useful. Its large angular field, combined with its large linear aperture, makes it possible to plot star fields very rapidly. It should also be possible to photograph meteors, a thing which has not yet been accomplished with any other lens. The illumination in the meteor image has been shown to depend on the aperture ratio and the quality of definition of the lens, and of course the large field is a necessity. There are other astronomical uses to which the lens will doubtless be put. There is, for instance, the problem of the shadow bands sweeping rapidly across the earth during a total eclipse of the sun. Before the cause of these, generally ascribed to diffraction, is fully understood, it will be necessary to obtain photographic, or rather kinematographic, records. The faintness and rapidity of the shadow bands calls for a very rapid lens. An attempt to obtain a photograph during the last eclipse of the sun unfortunately failed.

Fig. 5. *Iolanthe* at the Princes Theatre, London.

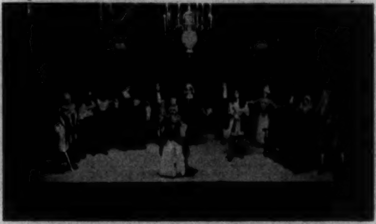


Fig. 6. Congreve's *The Way of the World* at the Lyric Theatre, Hammersmith.

DISCUSSION

Mr J. W. Perry: Mr Lee is to be congratulated upon the accomplishment detailed in this interesting paper. His results are of interest considered from the analytical standpoint and indicate what is successfully to be contended with in the struggle for complete mastery of the aberration question in general.

Mr T. Smith: Both the author and Messrs Taylor, Taylor and Hobson have earned the warmest thanks of the Society in communicating the details of performance of the new lens. As progress is made in the very difficult task of increasing the rapidity of photographic systems while maintaining a large field, it is of the greatest interest to many of our members to see an analysis of the results attained with a new system such as is given in the present paper. It will be observed that the author regards the algebraic computation of such a system as hopeless. That the difficulties in the way of such a feat are very great is obvious, and many will doubtless agree with the author's view until someone actually succeeds in overcoming these obstacles. Meanwhile opinions will no doubt differ as to the possibility of success by other means than ray tracing—which is what is evidently implied by algebraic methods. My own opinion, which I give for what it is worth, is that the prospects are less hopeless than is generally believed.

Mr Lee: The algebraic computation of aberrations has such advantages to the designer that it is to be hoped that the algebraic method advocated by Mr Smith, the scope of which he has so greatly extended by his own labours, will one day be able to compete with the trigonometrical, or partly algebraic, partly trigonometrical, and partly empirical methods of to-day. One of the advantages of the present method is the amount of more or less unskilled assistance that can be utilized in the work of design.

A WIDE-ANGLE STEREOSCOPE AND A WIDE-ANGLE VIEW-FINDER

By Col. L. E. W. van ALBADA, Amsterdam

[Communicated by Mr J. Rheinberg.]

MS. received, 29th April, 1924. Read and discussed, 12th June, 1924.

ABSTRACT. The paper describes a new type of lens system for use in wide-angle stereophotography and stereoscopy. The system gives a field of view of from 85° to 90° diagonally which is practically free from distortion. The many advantages of wide-angle stereoscopy over the ordinary type are discussed. A new and compact type of wide-angle view-finder is also described, and it is claimed that, with the exception of direct focussing, it yields all the advantages only obtainable hitherto with reflex cameras.

Introduction

I consider it a great honour to have the opportunity of explaining my methods of wide-angle stereoscopy to this Society. I take it for granted that you wholly agree with the well-known principles of orthostereoscopy as formulated first by Wheatstone and afterwards, among others, by Helmholtz. These principles, which form the basis of my lecture, depend upon the fact that, to obtain in the stereoscope as real an impression as possible, it is necessary that the stereoscopic pictures produce on the retinae of the observer's eyes images which, as regards size, form, and position, are similar to those actually seen by the operator when taking the pictures. The necessary conditions for this are as follows:

1. The lenses of the camera must be at the same distance apart as the eyes.
2. The corresponding points of a very distant object on the stereoscopic picture must be at the same distance apart as the nodal points of the stereoscope lenses.
3. The distance from the stereoscopic pictures to the nodal points of the stereoscope lenses must be equal to that from the plate to the nodal points of the camera lenses, that is, the pictures must be seen angle-true.

I wish to advocate the claims of wide-angle stereoscopy and to refute the misapprehensions which keep stereoscopy unjustly enclosed within far too narrow limits and prevent its natural development. I will show the optical means by which it is possible to extend the field of stereoscopic views considerably beyond the usual limits, which are imposed by the inability of the short focus magnifiers hitherto known to produce orthoscopic wide-angle views.

Experiment with Galilean Binoculars

Before proceeding, however, I would like to make a few remarks connected with physiological optics. Among the various means which enable us to get the impression of varying depth, I consider the alterations in the convergence of the eyes by far the most important; accommodation is only of secondary importance. I assured myself of this fact in the following way. I took a pair of Galilean binoculars, removed the eyepieces, placed the eyes at the foci of the objectives, and asked an assistant to move a stereoscopic picture to and fro along the axis of the instrument. The course of the rays was telescopic, and the images on the retinae did not alter their size, form, or position, consequently the convergence of the eyes remained the same and, when the picture was moved, only the accommodation altered with the distance. When the picture was being moved away, the objects on the picture seemed to increase, according to the apparent increase of the distance, and conversely, but as soon as it was stopped I could not decide whether it was near or distant. The same difficulty was experienced when, for example, the picture was divided into upper and lower halves placed at different distances.

With the above arrangement I accidentally discovered the fact that, when a strong light penetrates one eye sideways, the eye sees a white surface distinctly greenish, while the other eye, though not affected by the light, sees the same surface rosy, that is, in the complementary colour. It may thus be possible to see anaglyphs, executed in properly faint colours, stereoscopically without using glasses with the reversed colours, provided they are viewed with the side of the appropriate eye turned to the light.

GENERAL CONSIDERATIONS

Proceeding now to the question of obtaining orthostereoscopic views, we can imagine ourselves standing still in a room and looking through a window at the landscape outside. When we shut our right eye we can imagine the image of the landscape formed in our left eye to be projected on the window-pane and exactly copied with a pencil, and similarly with the image formed in our right eye when the left one is shut. Thus two drawings appear side by side. If now the landscape disappeared, we should see it unchanged in its former place because the two drawings would produce exactly the same images on the retinae as the landscape itself.

There is no reason, if we leave out of consideration the question of accommodation, why we should not approach the window so closely that we have a field of view possibly as wide as the natural field of view of the eye, reaching up to 120° or more. In stereophotography we must ensure that the pictures become identical with the drawings on the window-pane. This is very easy if we employ a stereo-camera, the lenses of which have the same focal length as the distance of the eyes from the window-pane, provided that their lateral separation is equal to the interocular distance, and that they are free from distortion, because the imaginary drawings on the window-pane are free from it.

It is often argued that photographs must not be taken with too large an angle of view because of the effect of exaggerated perspective and also because the principal object is not brought into such prominence with regard to the background and surroundings. This may be quite true in the case of single photographs, but in the case of stereo-pictures the argument does not hold.

One practical advantage of short-focus lenses, apart from their greater definition of depth, is that they are much smaller and lighter and so allow the cameras to be more compact and lighter. I would recommend the focal length to be not shorter than 2½ inches, as otherwise the effect of the grain of the plate comes in and prevents one obtaining good enlargements.

It is most desirable not to increase the distance between the lenses of a stereo-camera beyond the normal interocular distance of about 2½ inches, on account of the resulting deformation. In some cases, however, such as in detecting the relief of the surface of the earth as seen from an aeroplane, or even in astronomy in detecting differences in the distances of celestial bodies, the employment of considerably increased separation may have great advantages. On the other hand a smaller separation is of use in the case of very small and microscopic objects, for example, in the construction of stereoscopic views of small models for architectural purposes.

STEREOSCOPES

Mirror Stereoscope

The plainest and still the best stereoscope we know is Wheatstone's mirror stereoscope. Even for wide-angle stereoscopic pictures it might be of great use, permitting in any case a horizontal view of 60° and producing images unsurpassed in definition and general freedom from aberration. While deploring the fact that it has not found general appreciation, we may assume that this is only due to the circumstance that it requires separated pictures of considerable size.

Prism Stereoscope

Brewster's prism stereoscope, although technically and optically far inferior to the mirror stereoscope, has enjoyed an unequalled popularity all over the world.

Lens Stereoscope

If we want to take *instantaneous* stereoscopic photographs with lenses of shorter focal length than the distance of distinct vision, we *must* make use of lenses to supplement our defect of accommodation. The stereoscope lens has nothing else to do than give the retinal image its normal sharp definition. Practically this can be effected by making its focal length nearly the same as that of the camera lens.

Freedom from Distortion

The stereoscope lens must above all be free from distortion, as this imperfection exercises a most disturbing influence on the localization of the objects. To realize the significance of this fact we will observe that differences not greater than $\frac{1}{100}$ inch in the value of the distance apart of the two corresponding points of an object may produce a difference of depth of from 20 to 100 yards.

None of the lenses we commonly find in the so-called lens stereoscopes, not to speak of Brewster's prism stereoscope, are free from distortion. The defect is, of course, not very prominent when the lenses are only used for small angles of vision, but it increases progressively with the angle.

Fig. 1 shows the distortion produced by a prismatic lens of a Brewster's stereoscope and Fig. 2 the distortion produced by a common achromatic lens of good quality. The object in each case was a piece of squared paper, on which are marked the angles at which the squares were taken. The photographs were taken with reversed lenses, the stop thus coming inside the camera, so as to get the same course of the rays and the same distortion as in using the lenses in the stereoscope. Though in the subjective use of these lenses the definition is much better than indicated by these figures, because accommodation neutralizes the curvature of the image field, it is clear that the distortion even in the case of an achromatic lens is a prohibitive impediment to wider diagonal angles than 50°.

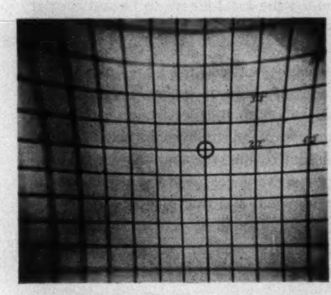


Fig. 1. Distortion of the prismatic lens of a Brewster's stereoscope (stop behind the lens).

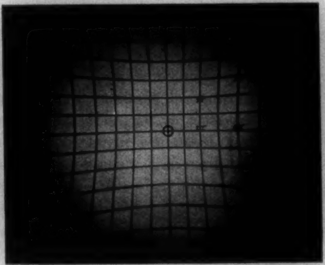


Fig. 2. Distortion of an achromatic stereoscope lens (stop behind the lens).

Verant Lens

For diagonal angles up to 57° there is, however, a lens which is free from distortion and gives, moreover, true perspective, namely the so-called Verant lens, designed by Prof. von Rohr and described before the Royal Photographic Society in 1903*. This lens will do excellently for all stereoscopic fields of vision at present used, but for wider angles the distortion becomes excessive.

Method of Correcting Distortion

It thus seems impossible to get a correct or orthoscopic view of a wide-angle stereo-picture of more than 60° diagonally, owing to lack of sufficiently orthoscopic magnifying glasses. There is, however, a well-known simple way of avoiding the distortion in the virtual stereoscopic image by purposely introducing the reversed distortion in the picture. If we take a wide-angle photograph with a landscape or meniscus lens with the stop in front, we get barrel-shaped distortion, as shown in Fig. 3, which was taken with the *same* lens as used for Fig. 2 and at the same distance from the squared paper. Now if we use the same lens in the stereoscope we shall see the picture quite correct with straight edges, because the rays travel in the opposite direction through the lens and thus suffer distortion of the opposite kind which exactly neutralizes that of the picture.

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* *Phot. Journ.* 43 (1903) 279.

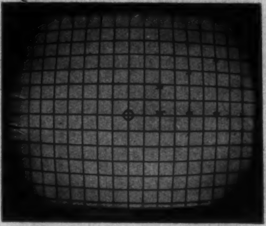

Fig. 3. Distortion of an achromatic stereoscope lens (stop in front of the lens).

This method has, however, several drawbacks, the principal one of which is the fact that landscape or meniscus lenses are not anastigmatically corrected and are therefore unsatisfactory for wide-angle photographs. Moreover, distorted pictures in direct vision always cause annoyance.

Considerations in the Design of a Wide-angle Stereoscope Lens

The desire to take wide-angle photographs with proper anastigmat lenses, and see the correct pictures with orthoscopic wide-angle magnifying glasses, made me try to find a system of stereoscope lenses which was sufficiently free from distortion over a field of nearly 90° diagonally. It may be of interest to describe how I finally succeeded in finding such a system.

It is, of course, well known that orthoscopy of a lens or system of lenses can only be obtained when the mean rays of all pencils of rays directed to one point on the principal axis of the system (the centre of the stop) *before* passing it also meet each other in one point on the axis *after* passing through the system. This can only take place when the system over its whole aperture is spherically corrected for the two conjugate points.

In the case of a magnifying glass orthoscopy is practically obtained when the narrow pencils of rays directed to the pupil of the eye after passing through the system are parallel to the directions they had before entering it. This, for example, is the case with a plane parallel glass plate, as is shown diagrammatically in Fig. 4, though the pencils of rays originating from one point do not exactly meet each other in one point again when, after passing through the plate, they are continued backwards. It was just this example which induced me to try to find a kind of magnifying glass plate.

Suppose we have a lens system as shown in Fig. 5. A point P on the axis is taken so that a ray, or narrow parallel pencil of rays, going from P to Q appears, after refraction at the flat surface 1, to come from a point M. If M is the centre of curvature of the spherical surface 2, the ray QR will not be refracted when it emerges from the first lens into the air, nor when it enters the second lens, provided the spherical surface 3 has the same centre of curvature M. Thus the ray, on reaching the second flat surface 4 at the point *R*, is refracted in a direction parallel to *PQ*. Thus the system is similar in action to a glass plate.

Moreover, when both lenses are made from the same kind of glass, there is no chromatic aberration, as the prisms at *Q* and *R* have the same angle but turned in opposite directions, nor is there any appreciable astigmatism because the rays pass normally across the spherical surfaces. The system has, however, a certain magnifying power, for the radius of curvature of surface 2 is smaller than that of surface 3.

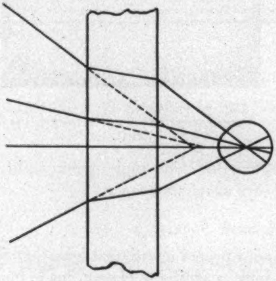

Fig. 4

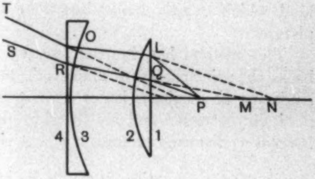

Fig. 5

This system would be an ideal one if *all* rays emerging from *P* were refracted by the plane surface 1 in directions which when continued backwards passed through *M*, but this is not the case. Thus a ray *PL* will, after refraction by the first lens, appear to come from a point *N farther* away than *M* and will consequently meet the spherical surface 3 at a point *nearer* to the axis than would be necessary to correct the deviation caused by the first lens. The consequence of this is that the angle between *RS* and *OT* is not so large as that between *PQ* and *PL*, so that the deviation of the pencils of rays towards the edge is somewhat under-corrected and towards the axis a little over-corrected.

Design ultimately adopted

Now there are various ways of obtaining better correction. One of these is to reverse the negative lens, increase its radius of curvature, and bring it nearer to the positive lens, as shown in Fig. 6. In this way we get a system which is practically corrected for distortion over a diagonal field of from 75° to 80° and shows but little chromatism. If a so-called achromatic positive lens is used, instead of the single lens, the correction for distortion may practically be carried up to 85° and 90°, while the system becomes almost entirely free from colour.

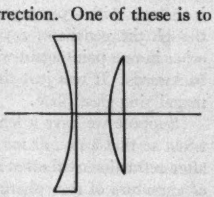

Fig. 6

The distortion, flatness of field, definition, and extent of the serviceable field obtained with such a system may be judged from a study of Figs. 7 and 8 which were taken with the same relative stop as Figs. 1, 2, and 3. Fig. 9 shows an architectural wide-angle photograph of 96° taken with a small stop, the lens being used as a landscape lens.

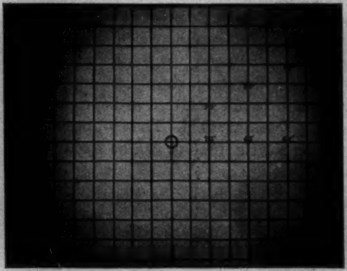

Fig. 7. Distortion of the wide-angle stereoscope lens (stop behind the lens).

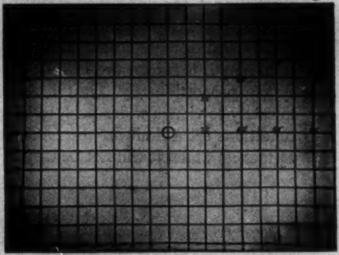

Fig. 8. Distortion of the wide-angle stereoscope lens (stop in front of the lens).

Fig. 9. Architectural photograph (angle 96°) with the wide-angle stereoscope lens used as landscape lens (stop in front of the lens).

Remarks as to Focal Length

With regard to the best focal length to employ the following remarks may be made. It would be desirable to make the focal length as short as possible but, in shortening it, the difficulties of getting a useful system increase progressively. We must bear in mind that the eye forms part of the complete optical system and that its size and distance from the nearest lens cannot be reduced. In shortening the focal length to widen the field of vision we cannot, therefore, simply reduce the system proportionately. On the contrary we should have to enlarge it disproportionately to view a wider field, or at least to keep the same field. These opposing conditions set a limit for the focal length (which for the present can be fixed at 2½ inches) which it will not be easy to reduce.

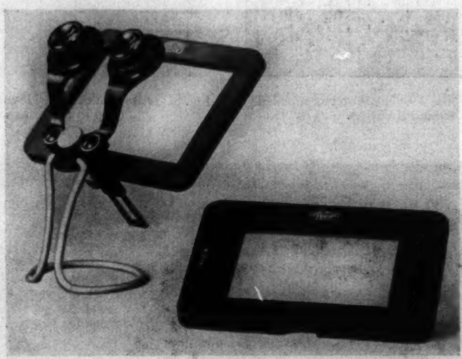

Fig. 10

Though the curvature of field of the system is very small, we have to place the picture far enough inside the focus on the principal axis to enable the margins to be seen distinctly, so that, for example, at a focal length of 2¾ inches, the real distance of the picture is reduced to 2½ inches. A system with a focal length of 2¾ inches allows a diagonal view of 85° to be seen at a distance of 2½ inches without displacing the centre of rotation of the eye (which is placed as close as possible to the lens) away from the axis of the system. When using the system as a key-hole, that is, moving the centre of rotation of the eye a little away from the axis and looking slantingly through the centre of the lens, a field of view of more than 100° can be obtained.

As these lenses are designed to give the widest possible view, it is evident that they must be used centrally, that is, their distance apart in the stereoscope must be equal to the observer's interocular distance and the eyes must be placed as close as possible to the lenses. The distance from the lenses to the picture accords with the emmetropic eye. Small hypermetropic and myopic deviations can be corrected by changing the distance to the picture; other anomalies can only be corrected by spectacles.

The complete stereoscope and a spare picture-holder are shown in Fig. 10. It will be seen that provision is made for altering the interocular distance and the distance of the picture from the lenses. (At the lecture a large number of stereoscopic pictures, taken with the new type of wide-angle stereoscopic camera, were exhibited in the special stereoscopes.)

WIDE-ANGLE VIEW-FINDER

Previous Types

I believe there is no part of photographic optics which is more neglected and undeveloped than that which deals with view-finders. The widely distributed so-called Brillant finder gives an image which is much too small, not too easy to find, and only visible in a definite position of the eye. It requires the camera to be kept at the distance of distinct vision beneath the level of the eye, giving thus the perspective of a child or a dwarf.

The Newton finder, consisting of a negative lens with a small sighting-vane, gives undoubtedly a clear reduced image at eye level, but it cannot give the exact limits of the image in its own plane, the boundary changing at every movement of the eye.

One of the best finders is the so-called Iconometer finder, which consists of a single vane behind a rectangular frame of wire that moves with the lens. It has, however, the disadvantages that the small frame is so near the eye that it appears very indistinct in the landscape and the eye must be at a certain position.

New Type

The drawbacks of these types of finder increase considerably for wide-angle photography and therefore I tried to construct a finder that was free from these objections. The principle of such a finder I discovered in the reversed common spy-glass, in which a reduced image appears in the stop of the eyepiece (Fig. 11).

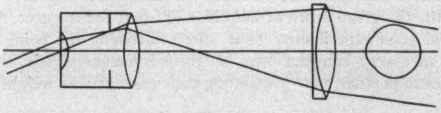

Fig. 11

With the aid of a stop of proper form and size we can limit the image field exactly. Moreover we can move the eye freely behind the front lens (that is, the objective) without the boundary or the definition of the field being altered. The reduction of the image field depends, as we know, on the magnifying power of the spy-glass as normally used.

For normal angles up to 60° a simple construction as shown in Fig. 12 will suffice. It turned out that Richard* had already designed a similar construction. Perhaps the quality of the images he obtained was not sufficiently good to use the device for determining the boundary of the field, because he only suggests its use to direct the centre of the plate on a definite object.

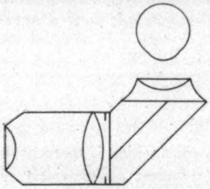
Fig. 12

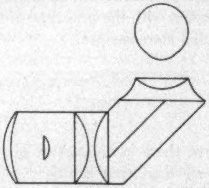
Fig. 13

For wide-angle purposes it was necessary to deviate considerably from the reversed spy-glass construction. Moreover special care had to be taken to eliminate distortion from so wide a field. The construction ultimately adopted is shown in Fig. 13. A view of the complete finder, as manufactured by Messrs Carl Zeiss, is given in Fig. 14. The back part, which only contains the eye lens and the reflecting roof prism (or a little mirror), can be turned about the axis of the front part for changing from the vertical to the horizontal position.

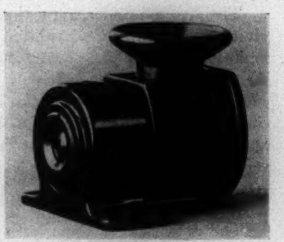
Fig. 14

This finder has shown itself extremely useful for wide-angle stereoscopic purposes. Its image is as clear as that of the naked eye and permits us to detect exactly whether a certain detail near the edge of the plate appears on the plate or not, with still more distinctness than the well-known reflex cameras can give. This property, combined with the great depth of definition of short focus lenses, makes the use of the ground glass superfluous. Thus, direct focussing not being required, the finder gives in a very compact form all the advantages which have made the reflex cameras so popular, notwithstanding their cumbersome weight and hood.

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* German Pat. 117015 (1889).

DISCUSSION

Prof. Cheshire: The author, whilst still satisfying the conditions that (1) stereoscopic photographs must be taken with an inter-objective distance equal to the inter-pupillary distance of the observer, and (2) each picture must be presented to the eye angle-true, has succeeded at the same time in very materially increasing the angular field of view. A distant landscape can be seen through a window either by standing on the hearthrug, or by sitting at the window—when looking at the author's slides, you felt that you were sitting at the window. The enhanced pleasure given by the wider angle of view is surprising and can only be realized by experiment.

Of course, the results that Col. van Albada has been successful in obtaining are made possible by his magnifying system, which is a very beautiful combination. Hitherto, the difficulty has always been to get viewing lenses with sufficient angle of view to match the taking lenses, but apparently the difficulty in future will be to get a photographic lens to match the viewing lens.

Col. van Albada is to be heartily congratulated upon having produced what is probably the highest and most charming expression of photographic stereoscopic art hitherto achieved.

Mr J. Rheinberg: I have greatly admired the demonstration of the wonderful results obtained by Col. van Albada's wide-angle stereoscopy and listened with great interest to his lecture, but what has struck me as much as anything is the very clear and simple way in which the lecturer has explained the principles on which he worked in designing his lenses for the purpose. To those who cannot in general easily follow the problems of lens construction and the complicated mathematics involved, it is a great pleasure to have the ideas leading up to the design of a lens system for a particular purpose, showing how it was evolved, set forth in so simple a manner.

Instr.-Lt.-Comdr. N. Langlands: This paper is of great interest because it opens out a new future for stereoscopic photography. I have often heard Prof. von Rohr express a very high opinion of Col. van Albada's ability to overcome mechanical and optical difficulties by ingenious devices. We have had a good example of that in the demonstration shown on this occasion.

Mr Wilfred E. L. Day: Has the lecturer adapted the principle of his lens to kinematography? If the wonderful results achieved by the new lens in the case of lantern slide production could be obtained with films, the value of screen productions would be enhanced enormously and a greatly added charm would be given to scenic films.

Dr J. S. Anderson: Is the new type of stereoscopic camera likely to be put on the market soon and, if so, what will be the approximate cost of the complete outfit?

Mr G. W. Gosling: Are the new lenses similar in their optical construction to the wide-angle magnifiers which bear the lecturer's name and which have been manufactured for some years by Messrs Zeiss?

Col. van Albada: I am very glad to note the interest taken in my lecture and demonstration and wish to thank the audience, especially those members who expressed their views in such kind words.

For the greater part the satisfaction is due to the beautiful pictures which some of my friends lent me for the demonstration.

I regret that, on account of an application for patents, I am not able to give fuller details of the optical data of the lens systems.

Although the principle of the wide-angle magnifier is not yet adapted to kinematography, it may be used for this purpose as well.

I have not yet put a special wide-angle stereo-camera on the market because several types, at present in use, can easily be made fit for this purpose. Special stereoscopes, provided with wide-angle magnifiers, will be put on the market by Messrs Zeiss next autumn at moderate prices, but I am not able to say what will be the approximate cost.

The wide-angle magnifiers which bear my name and have been manufactured for some years by Messrs Zeiss are not quite the same as those of the new type, which possess particular advantages.

BINOCULAR VISION AND THE STEREOSCOPIC SENSE

By R. J. TRUMP, B.A., B.Sc.

MS. received, 23rd May, 1924. Read and discussed, 12th June, 1924.

ABSTRACT. The convergence of the eyes, the visual focus, and the variation of these accommodations as the eyes sweep over the field do not play any vital part in the appreciation of distance. The recognition of a familiar type of perspective system is important, even to the extent of overriding the evidence of binocular parallax, when the two come into conflict. This may be shown by reversing the mounting of stereoscopic photographs, or by preparing stereo-diagrams in which the perspective is not in accord with the parallactic displacements.

Parallax, powerful in association with other factors, is uncertain, or even breaks down, when not assisted by perspective. There therefore appears to be no "mechanism" of stereoscopic vision dependent upon binocular parallax, but the perception of distance is a psychological function, involving the interpretation of all the details, similarities, and differences in the images in the two eyes, in the light of previous association and experience.

INTRODUCTION

IN his paper "Stereoscopy Re-stated"* read to this Society, Dr French arrives at the conclusion that stereoscopic perception involves a certain relationship between the axial convergence of the eyes and the focal accommodation. If this were so, it might be expected that, in examining an ordinary pair of stereoscopic pictures, the result would be appreciably modified by varying the distance of separation between corresponding points of the two component pictures, by which means the convergence of the eyes would be altered, but if the slide of the stereoscope were not moved, the focal accommodation would be unchanged. A certain amount of experience in stereoscopic photography having indicated that no noticeable modification of the observed effect was obtainable in this way, it appeared to be a matter of interest to look a little more closely into the matter, in order to ascertain, if possible, whether the independence of stereoscopic effect and the convergence of the optical axes of the eyes could be definitely established, and also to find how far convergence and optical focus were necessarily inter-related in stereoscopic appreciation of distance.

Observations with the Naked Eyes

A possible objection arises at the outset. Should a stereoscope, or indeed any artificial aid, be used at all in making observations upon binocular vision? The object of the instrument is, of course, to enable the eyes to take up an adjustment with their axes parallel, without requiring simultaneously an accommodation of focus suited for a near object. This type of focus is never required in actual life, and is therefore unfamiliar, and not easily secured. The stereoscope provides the extra focussing power which permits the object to be seen with the eyes focussed for infinity, a setting which is customarily associated with parallelism of the axes, and is therefore natural and comfortable. In view of the particular enquiry in mind, results obtained with a stereoscope might have been open to suspicion. It was therefore thought better to dispense with the instrument.

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* *Trans. Opt. Soc.* 24 (1922–23) 226.

It is well known that stereo-pictures may be seen with the unaided eyes. Upon looking at a stereo-pair, and imposing upon the eyes a "pensive" or "far-away" look—the description will no doubt be well understood—the axes of the eyes tend to become parallel, and each eye independently sees two pictures, making four in all, the two inner ones overlapping more or less. If the displacement can be carried far enough, the two images seen in the inner positions come into coincidence, but quite out of visual focus. It is then necessary to maintain the eyes fixed in position, while gradually altering the focus until clear vision is obtained. The stereoscopic effect is then apparent, just as in the stereoscope. The chief difficulty consists in holding the proper images in coincidence, while the visual focus is adjusted, as they show a marked tendency to break away, and one is left looking at the picture in the ordinary straightforward way. With practice, the necessary power can be developed.

A pair of stereo-prints were therefore taken, and clipped to a card, with corresponding points separated to a relatively small distance—about 4 cm. or so—and stereoscopic fusion obtained in this way. After a little practice, the separation was increased a little, and the operation was repeated until the necessary facility was obtained. The process is rather difficult when the pictures chosen are full of rather small detail, but much simpler when there is present a bold, strong object, such as, for instance, an isolated tree standing out in marked contrast against the sky. When the separation is somewhat large, and attention is fixed upon these prominent objects, they may be seen approaching each other up to a point, with seeming increasing reluctance. They then hesitate and stop, perhaps flutter a little, and then, as if definitely coming within a certain sphere of influence, they rush together and fuse. To obtain the best results, it is necessary to see that the vertical lines in the prints are set vertically, and that the images of corresponding points on the two sides are set at the same level, although no extreme accuracy is demanded. If then, in attempting to secure stereoscopic fusion, one image falls above or below the other, this error can be corrected by tilting either the head or the card in the appropriate direction.

With steady practice, gradually increasing the separation of the centres of the two pictures, quite a surprising degree of flexibility can be developed. It is possible to secure coalescence when the centres are separated distinctly more widely than the inter-ocular distance—as much as 85 mm. has been reached—while the eyes are focussed for a distance of no more than a foot or so. Just as with other unaccustomed physical exercise, however, it is necessary to follow out a careful system of training, and to be content with slow progress. Otherwise, as might be expected, serious and painful eye-strain will be experienced. Personal observation showed that, after a time, it was possible to snap the two images into fusion, almost without hesitation, over quite a wide range of separations, independently of the distance of the object from the eyes. In no case was there any essentially different appearance. Stereoscopic relief was neither more nor less apparent under these varying conditions. Moreover, it was even found that if the two pictures were reversed upon the mount, that is, if the left-hand picture were mounted upon the right, the two could still be fused and viewed by looking at them cross-eyed. This method was, indeed, habitually used in order to judge of the general effect of a negative before printing off.

It is now clear that stereoscopic relief is not sensibly affected over the most extreme possible range of ocular accommodation, from cross-eyed convergence to actual divergence of the optical axes of the eyes, the focussing adjustment being little if at all changed. It is difficult, therefore, to feel that the stereoscopic sense is due to any inter-relation between these adjustments. Nor, equally, can the absolute convergence of the eyes be very closely involved.

Observations with Galilean Binoculars

A complementary investigation now requires to be made, keeping the eyes parallel, and varying the focal accommodation. This was done by looking at the stereo-photographs through a pair of Galilean binoculars, with the lenses removed, the separation of the two fields being made equal to the interocular distance. Beyond a distance of about three feet, details of the pictures become difficult to distinguish. Once the general similarity of the results observed has been established by the eyes alone, it becomes permissible to insert the objectives in the tubes, by which means the range of focus can be greatly increased. In this case also it was found that stereoscopic fusion could be secured over the whole range of visual focussing power, with no apparent alteration in the relief observed.

Reversal of Normal Conditions

At this stage of practice it was found to be possible to produce a shift, with subsequent fusion, upon almost any object showing a repeat pattern—the panes of a leaded window, the pattern of a wall paper, the rails of a fence, the keys of a typewriter—obtaining an apparent coincidence after a shift of one, two, or more spaces in succession, according to the angular width of the pattern. Finally, by choosing a suitable object and background, a set of conditions was produced which was the exact reverse of the normal, namely, that an object in nearer visual focus appeared to be stereoscopically more remote. In other words, in order to restore clear vision in passing from the observation of one object to that of the other, it was necessary to increase the focal power of the eyes, while decreasing their convergence.

Experiment with Pins

The method adopted was quite simple. Two pins were pressed vertically into a piece of cardboard, and held between the eyes and the central vertical rail of a window, backed by the open sky. This choice of means has the merit of avoiding a number of factors which, by providing a psychological denial of the phenomenon which is being sought, greatly increase the difficulty of observing it. The pins, being considerably nearer to the eyes, become comparable in apparent diameter with the window rail, which facilitates proper placing. Incidentally the superfluous images can be thrown upon the curtain, and thus effectively lost. The chief advantage of the arrangement, however, lies in the fact that it does not offend one's sense of the correctness of things to see a distant object through a glass window. The employment of a screen, a wall, or other opaque object as a background adds very greatly to the difficulty of perception. The appearance of an object, seemingly located behind such a background, but remaining visible itself while obstructing the view of the background, is so incredible that psychological objections intervene, and perception invariably yields to the subconscious arguments of association and experience.

By the employment in this way of a pair of pins as an object, and regarding them as constituting a stereoscopic pair, two images of each may be formed by rotating the eyes outwardly, and the inner ones brought into coincidence and fused. It was found, when this was done, that a variation of the distance of the pins from each other, or from the eyes, did in fact cause the image to move in the manner stated by Wheatstone, with which Dr French disagrees. It is, in fact, quite possible to reduce the observation to quantitative terms, by carrying out a little rough and ready range-finding.

The results obtained, even with the crudest appliances, left no doubt that the apparent position in space of the complex image made up of the two pins does actually lie at the intersection of the axes of the two eyes, as originally stated by Wheatstone, and not at or near the object itself, as Dr French suggests. The difference is, however, not so great as it appears. If we imagine the optical conditions reversed, and the retinal images projected back into space, the conjugate images will fall, naturally, upon the pins. Any attempt to locate the composite image in space by a parallax method, *moving the head*, places it again on the pins. Keep the head still, and try to place the image with respect to surrounding objects, using only the parallax given in the two eyes, and it will be found where the optical axes of the eyes intersect. The whole appearance is, of course, an illusion. The complex image is a mental concept only, derived from the conjoint impression produced by two real objects. No such image, in any optical sense, exists at all, and so it cannot be properly placed anywhere; or, if desired, one may say that according to the point of view, it may appear to be located in either of two positions.

In view of the above experiences, it is now not unreasonable to suppose that, with sufficient practice, almost complete independence between the visual focus and the axial convergence of the eyes could be secured, and that, given suitable objects, the power of appreciating and interpreting parallax would still remain practically unimpaired. It will have been seen that any direct effect of the convergence of the eyes, as also of their focal accommodation, has been eliminated. Dove's observation—that a sense of distance is still retained, even when the object is only instantaneously illuminated—is fatal to the suggestion that any change in these accommodations, in passing from viewing one object to viewing another, is concerned. In order to leave no doubt upon the point, this experiment was repeated, with the aid of a Wimshurst machine, and fully confirmed.

Experiments with Various Diagrams

A doubt has also been thrown upon the sufficiency of binocular parallax alone to determine stereoscopic perception, when it comes into conflict with experience. It therefore became desirable to look more closely into the matter. In an attempt further to elucidate this problem, a number of pairs of stereo-diagrams were prepared, all based upon a fixed series of points, which were carefully pricked through several sheets of paper, so as to be identical in position throughout. The previous experiments had disclosed no apparent difference in the results, save of course in magnification, whether a stereoscope be used or not. As the instrument is a very useful aid to comfort, it may therefore well be used in observing these diagrams. So far as the more complete ones are concerned, the observations were confirmed by the use of the eyes alone. The simpler diagrams do not provide enough guidance to enable stereoscopic fusion to be secured at the rather extreme separations employed (about 80 mm.). A false fusion was usually obtained about two spaces short of the actual shift sought for, and stopped any further movement. In these cases, therefore, the stereoscope was necessarily used. The results obtained are given as personal; for although a general confirmation has been obtained in a fair number of cases, distinct individual differences have been recorded.

Fig. 1 shows the points A, B, C, D, E, F, G on one member of the pair of fields, all equally spaced upon the paper. The conjugate points A', B', etc. are unequally spaced, so as to give an apparent parallactic shift. The distances CC', EE', FF', BB', GG' DD', AA' all differ from each other successively by 0·5 mm., so that the apparent order of recession in space when observed through the stereoscope should be C, E, F, B, G, D, A. In this diagram the only feature present is this parallactic shift, free from any disturbing element, and the stereoscopic sensation should therefore be very strong. Actually, it was found very difficult to form any consistent impression at all. The pairs of dots, after being brought into coincidence, broke away and wandered, until perhaps a different coincidence altogether was obtained. It therefore became desirable to mark the centre dot D and its counterpart D' with a cross stroke, to ensure the proper coincidence. This done, the effect observed seemed to be that, when one pair, say D and D', are fused, neither C, C' nor E, E' are seen as single points; and similarly with most of the others. Upon turning the attention from one point to another, the two members of the second pair slide into each other without apparent difficulty, but seem to require a certain conscious effort to make them do so, and the movement as they join up is obvious. One can always, as it were, catch the doubled dot sliding into one. At the same time, its neighbours generally break up into two. The eyes very largely fail to place them in space in accordance with their parallactic separations.

In Fig. 2 the dots have precisely the same horizontal separations, but they have been placed at different heights, which are proportional to the parallactic separations of the respective points, in such a way that some sort of uniform, though crude, perspective pattern results. This provides distinct assistance in placing the dots. *C*, *B*, *A* and *E*, *F*, *G*, for instance, obviously form receding series, and *D* is behind both *C* and *E*. Moreover, they all constitute, in effect, a single "view," and it is not easy to see one point slide into its doublet as the attention sweeps over the field. The eyes are in some manner guided, so that the requisite change of convergence in passing from one point to another takes place to some extent automatically, as if there were a realization in advance as to what the necessary change was to be.

When the crude perspective is replaced by a set of less consistent ordinates, there is a distinct loss of the sense of relative distance. The points are more fluctuating and uncertain; doubling is apparent, and the sense of unity of the field is largely lost.

Fig. 1 may be modified by drawing vertical lines, all of the same length, through the respective points. The difficulty of identification again requires that the central line on each side shall be specially marked. The sense of distance obtained is neither immediate nor strong, though when found it is recognizable. The doubling of the lines, the uncertainty of the fusion, and the sliding of the doublets into each other are quite apparent.

In Fig. 3 vertical lines have been drawn in this way, but a little additional assistance is afforded by the long sloping lines, which appreciably help in placing the verticals. These now appear to lie in three definite and well-separated planes, *ADG*, *BF*, and *CE*. The tendency to doubling of the lines is not wholly gone, but it is so far reduced that the diagram does appear to constitute a single field, with some connection between the various lines.

Fig. 4 introduces a full and complete, though simple perspective system, consistent in every way with the parallactic separations. The effect is immediate and obvious. No doubt can arise as to the respective distances of the vertical lines of the series, which can be placed without hesitation in their correct order of distance. The stereoscopic perception is clear and unmistakable.

In Fig. 5, on the contrary, the perspective, as also the interruption of some of the vertical lines in the drawing, convey a wholly false suggestion. The parallactic displacements are such as exactly to contradict the impression conveyed by the lines of the drawing. Upon looking at it in the stereoscope, the eyes, if at best they do not seem to suggest that the whole constitutes merely a flat drawing, are deceived by the perspective, and lead us to regard the edge through *A* as the foremost, and that through *C* as the most remote. Binocular parallax should show the exact contrary, for the vertical lines are spaced exactly as in the last figure. At the same

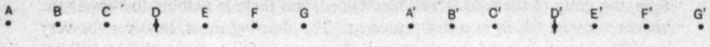

FIG. 1

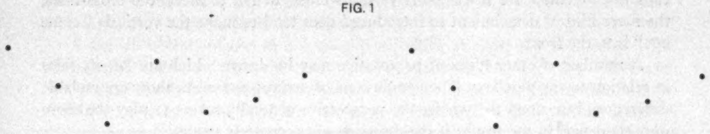

FIG. 2

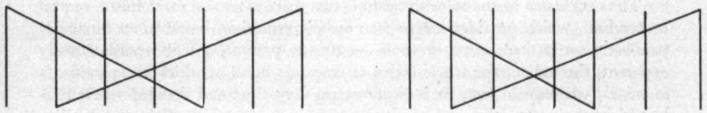

FIG. 3

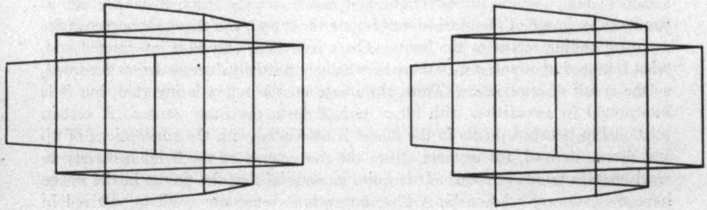

FIG. 4

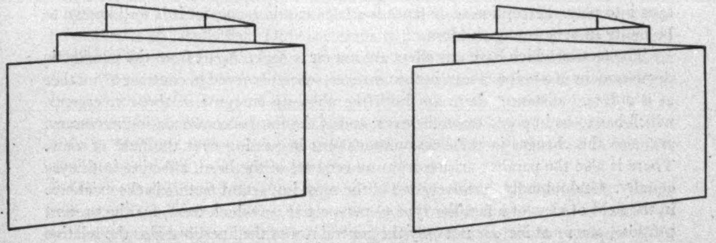

FIG. 5

time, the unity of the field is well maintained, and there is nothing to suggest the violent contrast which is actually present. The drawing must, however, be very carefully executed, for if the short verticals cross, or fail to meet, the cross-lines, the mere hint of detachment so introduced does tend to make the verticals "come out" into the front.

A number of other types of perspective may be drawn which are largely false in relation to the parallax. The conclusions of various observers show appreciable differences but, upon the whole, the perspective generally seems to play the more important part in the result, if the drawings are accurately made.

Discussion of Results

The conclusion seems to be irresistible that there is no real mechanism, optical or cerebral, which provides a true stereoscopic sensation, based upon binocular parallax. For if there were, it would be equally present, and its results equally apparent, throughout the whole series of diagrams in all of which the parallax is identical; whereas, in fact, its indications are very clear and decided where it is helped by the perspective, and uncertain, or even completely ineffective, where no such assistance is to be obtained.

This result should not, after all, be thought surprising. The eye is nothing but a camera and, organic imperfections and casual defects apart, it cannot lie. It produces an image of the outside world upon the retina, and the optic nerve carries a corresponding report to the brain. There and there only is it interpreted and, what is more important, it may there be wholly or partially disregarded or modified, as the result of experience. Thus, the image on the retina is inverted, but it is interpreted in association with ideas derived from the other senses. A certain relationship between points in the image is associated with the conceptions of up and down, derived, for instance, from the movements of the hand necessary to reach certain points in space. It is quite immaterial that the points in the image have the contrary relationship. The stereoscopic sense may well be derived in precisely the same way. At an early age the foveae, the only parts of the retina upon which clear vision is possible, develop into "corresponding points," if the term may be retained, from the attempt to see as clearly as possible, bringing both eyes into play. Perception of distance is a later acquirement, for it is well known to be faulty in very young children.

The factors which have any effect are not far to seek. Apart from the parallactic displacement of a point as seen in the two eyes, when observed in contrast to another at a different distance, there are both the absolute amounts of the convergence which has to be imposed upon the eyes, and of the focal accommodation necessary, and also the changes in these accommodations in passing over the field of view. There is also the parallax arising from movements of the head, affecting both eyes equally. Undoubtedly, however, one of the most important factors is the existence in the field of view of a familiar type of perspective, in which term, for the present purpose, one must include not only the general run of the lines, but also the relative apparent sizes of objects whose actual sizes are more or less known, the recognition of natural shading as associated with relief, and the knowledge that a more remote object is not visible through a nearer opaque one. In ordinary life, objects are seen invariably in association with others in a certain relationship, which in itself gives a powerful aid to the perception of distance.

A known object, placed in a known perspective system, can be located with considerable accuracy, when the visual parallax is also coming into play. Close one eye, and parallax is lost, but if the perspective remains, there is not much uncertainty. A mere movement of the head can, to a large extent, restore the position. Remove the perspective, as, for instance, over the sea and, although parallax remains, the judgment of distance of a landsman, working under unfamiliar conditions, is greatly impaired. An image formed in space by an optical system is quite difficult to place without a screen; and to take another instance, an absurdly incorrect judgment of distance may be made when a tiny spider hangs suspended in mid air upon its thread in front of the eyes. Remove both perspective and parallax, and the perception of distance may be wholly at fault. Hence the apparent great size of objects suddenly looming up through a fog, which appear large merely because their distances have been wholly misjudged.

In estimating distance, therefore, it would seem that binocular parallax does not play the supreme part with which it is usually credited. On the contrary, perspective, in the wide sense previously specified, appears to be more relied upon. Ocular adjustment may come in, but to a relatively minor degree. Whatever the eyes depict, however, the brain receives the impressions, and then proceeds to interpret them, adjusting them, as it were, in accordance with some psychological law of least squares, in order to arrive at the most probable conclusion. All so-called optical illusions are based upon this fact.

With an ordinary stereoscopic slide, the sense of reality of distance is very strong. Parallax is present, even, may be, exaggerated, owing to the fact that the lenses of the taking camera are perhaps separated more widely than the two eyes. Perspective, shading, relative sizes of the objects shown, and the mutual obstruction of the details are all in accord. Differential focussing of the eyes is, of course, wholly absent. If, however, the prints be transposed, parallax, which should completely pervert the apparent relief, is then in conflict with the other factors. In general, in such a case one experiences merely a sense of disappointment that the picture is unusually flat. Only occasionally does it appear that there is anything strange in the result, and it is rarely possible to detect the wholly false parallactic effect. The mind simply rejects the evidence of parallax when it leads to absurdity, although a foreknowledge of the effect to be expected does, at times, enable it to be detected, especially after some experience. In the majority of cases the verdict seems to be that, as the evidence as to solidity is conflicting, there is no solidity, but the picture is flat—as indeed it is known to be.

One other observation is perhaps particularly relevant at this point. It has frequently been remarked that if an ordinary photograph be looked at with one eye only, it takes on a distinctly more marked appearance of stereoscopic relief. This is particularly the case when the picture shows a strongly marked perspective.

It can now readily be explained why this is so. The correct perspective in all its aspects is present, but the picture is flat, and is known to be so. Looked at with both eyes, there is no parallax, and the real flatness is so emphasized that the mind is greatly hampered in endeavouring to reconstruct the original solid reality. With one eye closed, parallax could not be present, and its absence is not therefore remarked. The evidence which opposes the suggestion of the perspective is thus no longer there, and the imagination, which is of course in this case a willing agent, can form a much better appreciation of the original subject*.

CONCLUSIONS

The general conclusion arrived at is, therefore, that there is no real stereoscopic sense, based upon parallactic displacements between the images formed by the two eyes, but that these differences, frequently inconclusive in themselves, constitute merely one of the factors which, together with others at least as important, are utilized by the brain in forming its idea of distance. One of the most important of these other factors is the recognition of a perspective system, already familiar by association and experience. Further, just as one does not require to assume any special mechanism by which the inverted image upon the retina is erected before it is perceived, so one is led to think that there is nothing which can intelligibly be called a "mechanism" of stereoscopic vision, which is simply another manifestation of that extraordinary cerebral faculty which receives, collects, co-ordinates, and interprets all the sensory impulses which are continually reaching it, and employs them to build up a single unified conception of the world around us, even to the extent of setting off one sensation against another, in order to arrive at a conclusion which is justified by the sum of the evidence, when the latter is not wholly clear and unequivocal.

DISCUSSION

Instr.-Lt.-Comdr. N. Langlands: Mr Trump's paper is very interesting. His experiment with the pins is really similar to one of the first stereoscopic experiments of which we have any record—(Robert Smith—*A Compleat System of Opticks*, Cambridge, 1738). In making the statement that stereoscopic fusion of diagrams with the unaided eyes is more trustworthy than fusion with the aid of the stereoscope, Mr Trump has not considered the trouble which might arise from inferences drawn from the rotations of the eyes. Accommodation and convergence are of course of prime importance but, especially in "crossed" stereoscopic vision, we have to consider the Law of Listing.

The cases of perspective which the author has shown seem to be particular cases of the nine types of stereoscopic vision which have been exhaustively treated by Prof. von Rohr in his pamphlet on *Seeing with two eyes*. A less complete account is to be found in his *Binokularen Instrumente*—2nd edition, p. 256. No type of stereoscopic perspective could be found which did not belong to one of these classes.

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* Prof. Cheshire pointed out one among Col. van Albada's exhibits in which the foreground was best appreciated by using both eyes in the stereoscope (parallax) and the background by using one eye (perspective).

Mr Trump states that he has verified Dove's classical experiment by examining stereoscope diagrams illuminated by a short electric discharge. The important thing is to obtain quantitative and not qualitative results. I have conducted a long series of experiments and the results of these will be the subject of a paper in the near future.

Prof. Cheshire: I agree that this subject is one that should be approached with care and caution, because, as a matter of fact, a very large amount of important work has been done in connection with it since Wheatstone's days.

In the first place, there are one or two fundamental experiments of such great importance that they should always be borne in mind. One of the first of these is the description of what in effect is the stereoscopic range-finder by Robert Smith in 1738. In this experiment a pair of compasses with extended legs is held in front of the eyes of the observer. By proper adjustment of the angle between the legs a "W" can be seen when viewing a distant object. The middle leg produced by stereoscopic fusion can be made, by altering the angle between the legs of the compass, to stretch out to, and apparently touch, a distant object. Then, by measuring the distance between the compass points and the eyes, and the distance between the eyes and the compasses, the range can be determined.

Another important experiment is that by means of which Wheatstone showed that simple alteration in the convergence of the axes of the two eyes does not produce any marked stereoscopic impression. This experiment can be repeated with two parallel postal tubes placed before the eyes, with a vertical pin fixed at the farther end of one of them and a movable parallel pin in the corresponding end of the other; then, by looking through the tubes at a distant object and moving the adjustable pin across the line of sight, the pin produced by stereoscopic fusion can be seen moving up and down the line of sight. If, however, the second pin be fixed in the end of the tube, and an alteration in the angle of convergence be produced by an angular movement of one of the tubes in a horizontal plane about the centre of the eye, it will be found that a very considerable angular displacement of the tube is necessary to produce the impression of any displacement of the stereoscopic pin in the line of sight.

The third experiment that should be remembered is the test card for stereoscopic vision by Dr Pulfrich. Here a ruled graticule is placed before each eye. The distance between the corresponding lines in the two graticules is made to vary slightly in passing from one pair to another. The observer views these pictures and then proceeds to determine the order in which they appear to occur in the line of sight in the stereoscopic picture and he is finally called upon to plot a plan of the vertical lines as seen. In some of the finer tests the angle of convergence alters by not more than a few seconds in passing from one line to the other in the stereoscopic picture seen. Psychological difficulties in connection with range-taking are undoubtedly very disconcerting, especially when the observer is called upon to see a stereoscopic scale behind an opaque wall, but for one class of work the instrument is admirably adapted, namely, for use on air-craft.

Mr Trump: It was not the intention to suggest that more trustworthy results could be obtained with the eyes alone, but rather that the employment of auxiliary aid might have left a point for criticism. I have not seen Prof. von Rohr's pamphlet, but in referring to it Comdr. Langlands appears to have used the term "perspective" in a signification different to that given to it in this paper, where it is taken to describe all the features which are characteristic of a purely monocular representation of an object. In other words, perspective can be fully appreciated by one eye only, whereas parallax necessarily requires two. In this sense, therefore, "stereoscopic perspective" becomes a contradiction in terms.

With regard to Dove's experiment, it did not occur to me to imagine that, if stereoscopic vision was possible when the object was illuminated by an electric flash, as well as under ordinary observation, it could be lost under any other conditions. Comdr. Langlands' results in this connection will be awaited with interest. The point has been raised that, under instantaneous lighting, stereoscopic perception might be due to a mental conception of the change of convergence necessary to fix the various points of the field in succession. As any such estimate must be based upon the differences between the images formed in the two eyes, it is simpler to conclude that these differences are in themselves sufficient, than to consider that they must first be translated into terms of potential muscular effort. By analogy, one might say that it is more direct to judge of the height of a fence by eye, than by estimating the effort required to jump it.

Personal differences in the power of appreciating stereoscopic parallax are very great, and the faculty is one which can be much developed with practice; but in the case of no one of the limited number of observers who have been tried was there any failure to perceive it correctly in diagrams similar to those shown when an appropriate perspective was worked in. In the contrary case, failures were frequent, and occasionally almost complete.

A GENERAL SOLUTION OF THE FIRST ORDER ABERRATIONAL EQUATIONS

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ABSTRACT. The general equation by which the properties of the component parts of a compound optical system are determined is put into a new form. Solutions are obtained consisting of terms containing arbitrary constants together with any particular solution. The particular solution itself is the sum of two parts, one of which is independent of the corrections required in the complete instrument, while the other contains only the terms which express these special conditions. The arbitrary terms are given for a system of any number of components, and the particular solutions for systems of few components, the only cases which require special treatment.

Introduction

THE proper allocation of corrective duties to the various component lenses of a complex optical instrument formed the subject of an earlier investigation*, which contained all that was essential for the solution of the problem. The equations obtained were in the form of relations connecting the fundamental aberrational constants of the component lenses. Some of these, the number depending on the number of components, could be chosen arbitrarily, and the others were then determined uniquely from these relations. Though a solution in this form is theoretically unexceptionable, there are practical advantages in expressing it differently, the arbitrary degrees of freedom being represented by arbitrary constants. The expression of the conditions in this form is the aim of the present enquiry.

As in other cases, the Petzval condition is left out of account as its satisfaction, while mathematically of the greatest simplicity, actually depends upon the Gaussian plan of the instrument, which is assumed to be given. There remain the five primary aberrations to consider. As each component lens may be made to satisfy two conditions by the choice of suitable materials of construction and by the imposition of the proper form, the number of arbitrary constants will be $2n - 5$, where n is the number of component lenses.

The five primary aberrations will be determined by the five axial aberration coefficients† Δ_v, $v = 0, 1, 2, 3, 4$. In terms of these quantities the general relation of the earlier investigation may be written

$$\Sigma \frac{1}{\kappa_m} (\delta_0, \delta_1, \delta_2, \delta_3, \delta_4 \mathbin{)\!(} \lambda)_m \, (\mu)_m \, (\nu)_m \, (\xi)_m = \frac{1}{\kappa} (\Delta_0, \Delta_1, \Delta_2, \Delta_3, \Delta_4 \mathbin{)\!(} \lambda) \, (\mu) \, (\nu) \, (\xi) \quad \ldots (1),$$

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* *Trans. Opt. Soc.* 24 (1922–23) 168.

† *loc. cit.* p. 176, foot-note.

where δ_v is the coefficient for the mth component corresponding to Δ_v for the complete lens. By the special choice $S = 1$, $G = -1$ for the reference system of each component, $(\lambda)_m$ assumes the particular value

$$(\lambda_m + \lambda_{m-1},\ \lambda_m - \lambda_{m-1}) \qquad \text{......(2)},$$

and the coefficients under control are δ_3 and δ_4. In the particular case when the lens is thin the values are

$$\left.\begin{aligned} \delta_0 &= \delta_1 = 0 \\ \delta_2 &= \alpha_m = \tfrac{1}{2} + \tfrac{1}{3}\varpi_m \\ \delta_3 &= -\beta_m \\ \delta_4 &= \gamma_m \end{aligned}\right\} \qquad \text{......(3)}$$

in the notation extensively employed in earlier papers dealing with thin lenses. The problem for solution is thus the determination of general expressions for δ_3 and δ_4 in equation (1).

A new form for the general equation

Before proceeding to a solution it will be noted that this equation can be put into a more convenient form. For consider the table of aperture intersections*

$$\begin{matrix} h_{01} & h_{02} & h_{03} & \cdots & h_{0n} \\ h_{11} & h_{12} & h_{13} & \cdots & h_{1n} \\ h_{21} & h_{22} & h_{23} & \cdots & h_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ h_{n1} & h_{n2} & h_{n3} & \cdots & h_{nn} \end{matrix}$$

which may be constructed either in the way already described from the traces of two paraxial rays or alternatively from

$$h_{p,p} = h_{p,p+1} = 1$$

with

$$\left.\begin{aligned} h_{p,q} - h_{p-1,q} &= \kappa_p\,[pq] \\ h_{q,p} - h_{q+1,p} &= \kappa_q\,[pq] \\ h_{p,q} - h_{p,q+1} &= t_q \kappa_{p+1,q} \\ h_{q,p+1} - h_{q,p} &= t_p \kappa_{p+1,q} \end{aligned}\right\} (p < q) \qquad \text{......(4)},$$

or

where the apparent separation between lenses p and q is denoted by $[pq]$, so that

$$[pq] = -\frac{\partial^2 \kappa_{p,q}}{\partial \kappa_p \partial \kappa_q}, \quad p \not> q - 1.$$

Particular values are

$$[pp] = 0; \quad [p, p+1] = t_p; \quad [p, p+2] = t_p + t_{p+1} - t_p t_{p+1} \kappa_{p+1}.$$

The λ's, μ's, etc. are identified with selected columns of the above h's. Now define new quantities H_p and H_{mp} by the equations

$$H_p = \frac{h_{n,p} + h_{0,p}}{h_{n,p} - h_{0,p}} \qquad \text{......(5)},$$

and

$$H_{mp} = \frac{h_{m,p} + h_{m-1,p}}{h_{m,p} - h_{m-1,p}} \qquad \text{......(6)}.$$

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* *loc. cit.* p. 177.

The values in certain cases are evidently

$$H_{pp} = \infty\ ;\ H_{p,\,p-1} = 1 - \frac{2}{\kappa_p t_{p-1}}\ ;\ H_{p,\,p+1} = \frac{2}{\kappa_p t_p} - 1.$$

Instead of calculating the h's initially, the H's may be found directly using the relations

$$\left.\begin{aligned} H_{pq} - H_{pr} &= \frac{2\,[qr]}{\kappa_p\,[pq]\,[pr]} \\ H_{qr} - H_{qp} &= \frac{2\,[pr]}{\kappa_q\,[pq]\,[qr]} \\ H_{rp} - H_{rq} &= \frac{2\,[pq]}{\kappa_r\,[pr]\,[qr]} \end{aligned}\right\}(p < q < r) \qquad \text{......(7).}$$

It will be noted that the sign of the H differences is determined by the cyclical order of the suffixes. Now from (2), if λ and μ are identified with columns p and q of the above table respectively,

$$\begin{aligned} (\lambda)_m &= (h_{m,\,p} + h_{m-1,\,p},\, h_{m,\,p} - h_{m-1,\,p}) \\ &= (H_{mp}, 1)\,\kappa_m\,[mp] \end{aligned}$$

by (4) and (6), so that by (7)

$$\begin{aligned} (\lambda)_m\,(\mu)_m &= (H_{mp}, 1)\,(H_{mq}, 1)\,\kappa_m^2\,[mp]\,[mq] \\ &= \frac{(H_{mp}, 1)\,(H_{mq}, 1)}{H_{mp} - H_{mq}}\,.\,2\kappa_m\,[pq], \end{aligned}$$

with similar expressions of like form for $(\nu)_m$ and $(\xi)_m$. The corresponding quantities for the complete lens may be treated in a similar way. On removing the common factors, (1) then takes the form

$$\Sigma\,\frac{\kappa_m\,(\delta_0, \delta_1, \delta_2, \delta_3, \delta_4)\!\!(H_{mp}, 1)\,(H_{mq}, 1)\,(H_{mr}, 1)\,(H_{ms}, 1)}{(H_{mp} - H_{mq})\,(H_{mr} - H_{ms})}$$

$$= \frac{\kappa\,(\Delta_0, \Delta_1, \Delta_2, \Delta_3, \Delta_4)\!\!(H_p, 1)\,(H_q, 1)\,(H_r, 1)\,(H_s, 1)}{(H_p - H_q)\,(H_r - H_s)} \qquad \text{......(8).}$$

It will be observed that the grouping of the brackets is always such that p and q are different integers, and similarly r and s are different.

When m is equal to one of the integers p, q, r, or s, the H with the equal suffixes becomes infinite. This number appears in both numerator and denominator and its coefficients alone are to be retained. Thus if $m = p = r$, q and s being different, this particular term on the left of (8) becomes

$$\kappa_p\,(\delta_0, \delta_1, \delta_2)\!\!(H_{pq}, 1)\,(H_{ps}, 1)$$

and if $m = p$, while q, r, and s differ from p, the contribution to the sum is

$$\frac{\kappa_p\,(\delta_0, \delta_1, \delta_2, \delta_3)\!\!(H_{pq}, 1)\,(H_{pr}, 1)\,(H_{ps}, 1)}{H_{pr} - H_{ps}}.$$

It is easily seen that equation (8) is strictly algebraic, the signs in all cases following automatically from the definitions.

The coefficients of the general constants

A particular solution of equations (8) may be found without difficulty. From this solution it is possible to derive every other solution by giving suitable values to the arbitrary constants. The matter of chief moment is to determine the coefficients to be combined with these constants in the expressions for δ_3 and δ_4 for the various components. The problem is obviously to find $(2n-5)$ sets of quantities which satisfy the relation

$$\Sigma \frac{X_m + Y_m (H_{mp} + H_{mq} + H_{mr} + H_{ms})}{(H_{mp} - H_{mq})(H_{mr} - H_{ms})} = 0 \qquad \text{......(9),}$$

however p, q, r, and s may be chosen. From the form of the equation it may be anticipated that Y will be of the first and X of the second degree in the H's. Take an arbitrary closed cycle of integers *abcd...k*, arranged in any order, none of which exceeds n. Any particular integer a follows another k and is succeeded by a third b. The relations of b and k to a may be expressed by the positive and negative signs respectively.

In the condition (9) substitute

$$\left.\begin{aligned} -Y_a &= H_{ab} - H_{ak} \\ X_a &= 2\,(H^2{}_{ab} - H^2{}_{ak}) \end{aligned}\right\} \qquad \text{......(10),}$$

and similarly for other members of the cycle *abcd...k*, and zero in other cases.

Consider the terms which involve the two suffixes a and b. Since

$$\begin{aligned} 4\,(H^2{}_{ab} - H^2{}_{ak}) - 2\,(H_{ab} - H_{ak})(H_{ap} + H_{aq} + H_{ar} + H_{as}) \\ = (2H_{ab} - H_{ap} - H_{aq})(2H_{ab} - H_{ar} - H_{as}) \\ - (2H_{ak} - H_{ap} - H_{aq})(2H_{ak} - H_{ar} - H_{as}), \end{aligned}$$

the terms that have to be considered are

$$\frac{(2H_{ab} - H_{ap} - H_{aq})(2H_{ab} - H_{ar} - H_{as})}{(H_{ap} - H_{aq})(H_{ar} - H_{as})}$$

and

$$-\frac{(2H_{ba} - H_{bp} - H_{bq})(2H_{ba} - H_{br} - H_{bs})}{(H_{bp} - H_{bq})(H_{br} - H_{bs})},$$

and it will be shown that their sum vanishes identically. By considering all the pairs as they occur in the circuit it follows that the addition of any common multiple of the X's and Y's to their respective δ_4's and δ_3's makes no difference to the sum on the left of (8). Let

$$\frac{H_{ab} - H_{ad}}{H_{ac} - H_{ad}} = (abcd) \qquad \text{......(11).}$$

By (7)

$$\begin{aligned} (abcd) &= \frac{[ac]\,[bd]}{[ab]\,[cd]} \\ &= (badc) \end{aligned} \qquad \text{......(12),}$$

and the equality of

$$[(abpq) - (abqp)]\,[(abrs) - (absr)]$$

and

$$[(bapq) - (baqp)]\,[(bars) - (basr)],$$

that is to say the vanishing of the sum of the *ab* terms, at once follows.

The foregoing formulae will evidently be of particular value in dealing with special problems, such as the determination of the alterations necessary in two other selected components to balance a certain change which it is desired to introduce in a third lens. The coefficients (10) give the necessary changes without further calculation.

The independence of circuits

To ensure that the formal solution in any particular case is quite general, care must be taken to see that the circuits are all independent, that is, that the arbitrary terms are not expressible with a smaller number of arbitrary constants. In the cases when n is small this is comparatively simple. For example, with triangular circuits the following solutions may be taken:

$$n = 3; \quad (123).$$
$$n = 4; \quad (123), (234), (341).$$
$$n = 5; \quad (135), (125), (245), (234), (134).$$

Other circuits may be constructed from these; for example, when $n = 4$ the fourth possible cycle (412) is given by the relation

$$(123) + (341) = (234) + (412).$$

Examples of quadrilateral circuits are

$$n = 4; \quad (1234), (1324), (1342).$$
$$n = 5; \quad (1234), (2345), (3451), (4512), (5123).$$

As an instance of a group of five quadrilaterals for $n = 5$ which are not independent, the set

$$(1254), (2315), (3421), (4532), (3143)$$

may be taken.

The examination for independence may be facilitated by both algebraical and geometrical means. The methods below yield in general a larger number of circuits than the number of independent constants in the equations, but they are useful in rejecting a large number of circuits which are related in simple ways.

Taking the algebraical method first, suppose that the set of circuits (123), (1473), (2356), (1267), (247), ... is to be examined. Take each pair of integers in their natural order and enter opposite them undetermined quantities for each circuit. Thus for the first circuit enter A opposite 12 and 23 and $-A$ opposite 13. Similarly for the second circuit enter B opposite 14 and 47 and $-B$ opposite 13 and 37; and so on. The entries opposite each pair of integers are to be added together. The circuits are not independent if all these sums can be made to vanish simultaneously while any of the quantities $A, B, C, \ldots$ remain finite.

The geometrical method depends on the selection of component areas of a polygon of n sides in such a way that no other area, whether the whole or a part, can be built out of these parts in more than one way. The angular points may be denoted by the numbers 1, 2, 3, ... n and the area formed by joining these points in this order by (123...n). Attention may be confined to convex polygons. When

$n = 3$ the only possible area is the triangle (123), and this is the complete solution as given above. When $n = 4$ the sum of the areas (123) and (341) is identical with the area (1234) of the whole polygon, and the same applies to the sum of (234) and (412). There is therefore one relation between the four possible triangular components, leaving three independent circuits. When $n = 5$, five independent quadrilaterals can be obtained by suppressing each of the angular points in turn, but to enable every area to be expressed in terms of the components, the complete polygon or a triangle or a concave quadrilateral must be added, giving a total of six components. Again there is one relation between the four triangles contained by each of the five convex quadrilaterals, but that given by the fifth is derivable from the other four. There are therefore four relations among the complete set of ten triangles, leaving six triangular circuits for consideration. Fig. 1 illustrates a set of six triangles which satisfies the conditions. When $n = 6$, 15 quadrilaterals can be constructed. Of these, if the points are taken in order at the vertices of a regular hexagon, 3 are rectangles such as (1245), 6 are kite-shaped figures such as (1235), and 6 are the halves of the hexagon completed by a diagonal, as (1234).

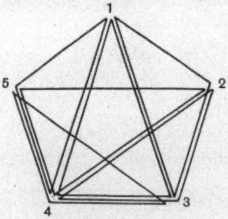

Fig. 1

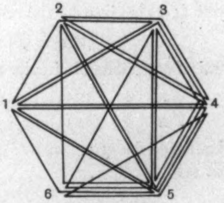

Fig. 2

Since the last six may be combined in pairs to yield the complete hexagon, there are two relations connecting them, and they are equivalent to only four independent elements. Also the combination of two opposite kite-shaped figures yields the same result as a pair of rectangles. It follows that this method reduces the number of possible quadrilateral circuits from 15 to 10. Again there are 20 possible triangles, and since each independent quadrilateral gives an independent relation among them, the number of triangular circuits is reduced from 20 to 10, the same number as with the quadrilaterals. Fig. 2 illustrates a set of triangles satisfying this condition. It will be noted that the final number of circuits in this case exceeds by three the number of general constants in the solution of the equations. If all ten are included it is certain that no degree of freedom has been neglected.

As Figs. 1 and 2 illustrate, $\frac{1}{2}(n-1)(n-2)$ triangular circuits satisfying these geometrical conditions may be selected by taking the $n-2$ triangles having 1 as vertex and consecutive points of the polygon as the ends of their bases, together with $n-3$ triangles having 2 as vertex and the joins of consecutive points for bases excluding the point 1, and generally $n-p-1$ triangles of the form $(p, q, q+1)$

where q takes in turn all values from $p + 1$ to $n - 1$. This gives $\frac{1}{2}(n-3)(n-4)$ more circuits than there are general constants in the solution, and the further reduction must depend upon relations between the H's. It will be noted that the number of finite H's is $n^2 - n$, of which only $2(n-1)$ are independent, since this is the number of independent h's. Thus there are $(n-1)(n-2)$ relations connecting the H's.

Particular solutions

Although, as has been remarked, the determination of particular solutions offers no difficulty, it may be of interest to include examples covering systems of the complexity found in photographic lenses, particularly as it appears that it is customary for designers to reach such a solution by trial and error instead of by a direct expression. These particular solutions may be conveniently divided into two parts; one yields the results for the case in which the axial aberration is entirely removed, that is when all the Δ's are zero; the other gives the corrections necessary when finite values are to be secured for the Δ's, and depends only on the special corrections required in the particular problem.

The first part of the solution will be considered first. The letters p, q, r, s denote different lens numbers, and the solutions are written for lens p, the other cases following from symmetry.

Three lenses.

$$\kappa_p(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{pq}, 1)(H_{pr}, 1)(H_{pq} + H_{pr}, 2)$$
$$= (H_{pq} - H_{pr})\{\kappa_q(\delta_0, \delta_1, \delta_2 \between H_{qr}, 1)(H_{qp}, 1) - \kappa_r(\delta_0, \delta_1, \delta_2 \between H_{rp}, 1)(H_{rq}, 1)\} \quad \text{......(13)},$$

$$\kappa_p(\delta_0, \delta_1, \delta_2, 0, \delta_4 \between H_{pq}, 1)(H_{pr}, 1)(H_{pq} + H_{pr}, 2)(3H_{pq} + 3H_{pr}, -2)$$
$$= 4(H_{pq}^2 - H_{pr}^2)\{\kappa_q(\delta_0, \delta_1, \delta_2 \between H_{qr}, 1)(H_{qp}, 1) - \kappa_r(\delta_0, \delta_1, \delta_2 \between H_{rp}, 1)(H_{rq}, 1)\}$$
$$+ (H_{pq} - H_{pr})^2\{4\kappa_q(\delta_0, \delta_1, \delta_2 \between H_{qr}, 1)^2 + 4\kappa_r(\delta_0, \delta_1, \delta_2 \between H_{rq}, 1)^2$$
$$- \kappa_p(\delta_0, \delta_1, \delta_2 \between H_{pq}, 1)(H_{pr}, 1)\} \quad \text{......(14)}.$$

When the components are thin these expressions take the forms

$$\kappa_p\{2\beta_p - 3\alpha_p(H_{pq} + H_{pr})\} + (H_{pq} - H_{pr})(\kappa_q\alpha_q - \kappa_r\alpha_r) = 0 \quad \text{......(15)},$$

and $$\kappa_p\{\tfrac{1}{2}\gamma_p - \alpha_p(H_{pq}^2 + H_{pq}H_{pr} + H_{pr}^2)\} + (H_{pq} - H_{pr})(\kappa_q\alpha_q H_{pq} - \kappa_r\alpha_r H_{pr}) = 0 \quad \text{......(16)}.$$

These expressions are easily derived from the equations.

Four lenses.

When there are four components the case in which p, q, r, s in (8) are all different shows that the δ_3's may be chosen to satisfy

$$\kappa_p(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1) = 0 \quad \text{......(17)}.$$

The remaining four conditions are of the type

$$\frac{\kappa_p(\delta_0, \delta_1, \delta_2, \delta_3, \delta_4 \between H_{pq}, 1)^2 (H_{pr}, 1)(H_{ps}, 1)}{(H_{pq} - H_{pr})(H_{pq} - H_{ps})} + \kappa_q(\delta_0, \delta_1, \delta_2 \between H_{qr}, 1)(H_{qs}, 1) - \frac{\kappa_r(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{rq}, 1)^2 (H_{rs}, 1)}{H_{rq} - H_{rs}} - \frac{\kappa_s(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{sq}, 1)^2 (H_{sr}, 1)}{H_{sq} - H_{sr}} = 0.$$

Now

$$\begin{aligned} &\kappa_p(\delta_0, \delta_1, \delta_2, \delta_3, \delta_4 \between H_{pq}, 1)^2 (H_{pr}, 1)(H_{ps}, 1) \\ &+ \kappa_p(\delta_0, \delta_1, \delta_2, \delta_3, \delta_4 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1)(H_{pq} + H_{pr} + H_{ps}, -1) \\ &\quad = (2H_{pq} + H_{pr} + H_{ps})\,\kappa_p(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1) \\ &\quad = 0, \end{aligned}$$

in consequence of (17). Also by the same formula

$$\kappa_r(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{rq}, 1)^2 (H_{rs}, 1) = (H_{rq} - H_{rp})\,\kappa_r(\delta_0, \delta_1, \delta_2 \between H_{rq}, 1)(H_{rs}, 1).$$

Moreover $$\frac{H_{rq} - H_{rp}}{H_{rq} - H_{rs}} = (rpsq) = (prqs) = \frac{H_{pr} - H_{ps}}{H_{pq} - H_{ps}}.$$

Substituting this value and dealing with the last term in a similar way gives

$$\begin{aligned} &\kappa_p(\delta_0, \delta_1, \delta_2, 0, \delta_4 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1)(H_{pq} + H_{pr} + H_{ps}, -1) \\ &\quad = (H_{pq} - H_{pr})(H_{pq} - H_{ps})\,\kappa_q(\delta_0, \delta_1, \delta_2 \between H_{qr}, 1)(H_{qs}, 1) \\ &\qquad + (H_{pr} - H_{ps})(H_{pr} - H_{pq})\,\kappa_r(\delta_0, \delta_1, \delta_2 \between H_{rs}, 1)(H_{rq}, 1) \\ &\qquad + (H_{ps} - H_{pq})(H_{ps} - H_{pr})\,\kappa_s(\delta_0, \delta_1, \delta_2 \between H_{sq}, 1)(H_{sr}, 1) \qquad \text{......(18).} \end{aligned}$$

The simplicity of the general results in this case makes it unnecessary to deal specially with the case when the components are thin.

Five lenses.

Since there are five degrees of freedom present one possible choice is to make all the δ_3's zero. By giving p, q, r, s different values the condition becomes an equation for the value of δ_4 for the remaining component. The solution then takes the form

$$\delta_3 = 0 \qquad \text{......(19),}$$

$$\begin{aligned} &\frac{\kappa_p(\delta_0, \delta_1, \delta_2, 0, \delta_4 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1)(H_{pt}, 1)}{(H_{pq} - H_{pr})(H_{ps} - H_{pt})} \\ &\quad + \frac{\kappa_q(\delta_0, \delta_1, \delta_2, 0 \between H_{qr}, 1)(H_{qs}, 1)(H_{qt}, 1)}{H_{qs} - H_{qt}} \\ &\quad - \frac{\kappa_r(\delta_0, \delta_1, \delta_2, 0 \between H_{rs}, 1)(H_{rt}, 1)(H_{rq}, 1)}{H_{rs} - H_{rt}} \\ &\quad + \frac{\kappa_s(\delta_0, \delta_1, \delta_2, 0 \between H_{st}, 1)(H_{sq}, 1)(H_{sr}, 1)}{H_{sq} - H_{sr}} \\ &\quad - \frac{\kappa_t(\delta_0, \delta_1, \delta_2, 0 \between H_{tq}, 1)(H_{tr}, 1)(H_{ts}, 1)}{H_{tq} - H_{tr}} = 0 \qquad \text{......(20).} \end{aligned}$$

This solution is of an entirely different type from those given for three and four lenses, and, since each component, if of a single glass, is equiconvex or equiconcave, it illustrates that a practicable form for manufacture may readily be secured for the particular solution. It may be of interest to add a solution of the earlier type. Its derivation offers no difficulty.

Let δ_3 be given by

$$\begin{aligned}\kappa_p(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1)\\ + \kappa_p(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{pr}, 1)(H_{ps}, 1)(H_{pt}, 1)\\ + \kappa_p(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{ps}, 1)(H_{pt}, 1)(H_{pq}, 1)\\ + \kappa_p(\delta_0, \delta_1, \delta_2, \delta_3 \between H_{pt}, 1)(H_{pq}, 1)(H_{pr}, 1) = 0,\end{aligned}$$

that is by

$$\kappa_p(0, \delta_0, 2\delta_1, 3\delta_2, 4\delta_3 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1)(H_{pt}, 1) = 0 \quad \text{......(21)}.$$

By proceeding as in the case of four lenses it at once follows that

$$\begin{aligned}&4\kappa_p(\delta_0, \delta_1, \delta_2, 0, \delta_4 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1)(H_{pt}, 1)\\ &- (H_{pq} + H_{pr} + H_{ps} + H_{pt})\,\kappa_p(0, \delta_0, 2\delta_1, 3\delta_2, 0 \between H_{pq}, 1)(H_{pr}, 1)(H_{ps}, 1)(H_{pt}, 1)\\ &+ \Sigma (H_{pq} - H_{pr})(H_{pq} - H_{ps})\,\kappa_q(\delta_0, \delta_1, \delta_2 \between H_{qr}, 1)(H_{qs}, 1) = 0 \quad \text{......(22)},\end{aligned}$$

where the summation denotes that q is to receive in turn all values but p, and that r and s represent each pair of values excluding p and q.

Evidently cases where there are still more components may be treated in a similar way.

The special conditions

It remains to insert the special terms by which particular types of correction are obtained when the system is not to be designed for the absence of axial aberration. Provision against error is secured by introducing these conditions in a symmetrical form. Owing to the degrees of freedom always present when there are more than two components this can be done in a number of ways. By analogy with the discussion of the previous section it may be inferred that if a symmetrical form is secured for the terms affecting the δ_3's, symmetry for the δ_4's will follow.

Three lenses.

The correction to $\kappa_p\delta_3$ for lens p given by equation (13) is

$$-\frac{(H_{pq} - H_{pr})\,H_p\,\kappa(\Delta_0, \Delta_1, \Delta_2, \Delta_3, \Delta_4 \between H_p, 1)(H_q, 1)(H_r, 1)(H_q + H_r, 1)}{(H_p - H_q)(H_q - H_r)(H_r - H_p)} \quad (23),$$

and the correction to $\kappa_p\delta_4$ is

$$\begin{aligned}&\frac{2(H_{pq}^2 - H_{pr}^2)\,H_p\,\kappa(\Delta_0, \Delta_1, \Delta_2, \Delta_3, \Delta_4 \between H_p, 1)(H_q, 1)(H_r, 1)(H_q + H_r, 1)}{(H_p - H_q)(H_q - H_r)(H_r - H_p)}\\ &\qquad + \frac{(H_{pq} - H_{pr})^2\,\kappa(\Delta_0, \Delta_1, \Delta_2, \Delta_3, \Delta_4 \between H_q, 1)^2(H_r, 1)^2}{(H_q - H_r)^2} \quad \text{......(24)}.\end{aligned}$$

Four lenses.

The formulae having the fullest symmetry are not concise. A solution may therefore be based upon a particular group p, q, r. The expressions are then of the form:

for p, q, r group; correction to $\kappa_p\delta_3$ is zero;

correction to $\kappa_p\delta_4$ is

$$\frac{(H_{pq}-H_{ps})(H_{pr}-H_{ps})}{(H_q-H_s)(H_r-H_s)}\kappa(\Delta_0,\Delta_1,\Delta_2,\Delta_3,\Delta_4 \between H_q,1)(H_r,1)(H_s,1)^2 \quad \text{...(25)},$$

and for s; correction to $\kappa_s\delta_3$ is

$$\frac{H_{sp}-H_{sq}}{(H_p-H_q)(H_s-H_r)}\kappa(\Delta_0,\Delta_1,\Delta_2,\Delta_3,\Delta_4 \between H_p,1)(H_q,1)(H_r,1)(H_s,1) \quad \text{...(26)};$$

correction to $\kappa_s\delta_4$ is

$$-\frac{(H_{sp}+H_{sq}+2H_{sr})(H_{sp}-H_{sq})}{(H_p-H_q)(H_s-H_r)}\kappa(\Delta_0,\Delta_1,\Delta_2,\Delta_3,\Delta_4 \between H_p,1)(H_q,1)(H_r,1)(H_s,1)$$
$$+\frac{(H_{sp}-H_{sr})(H_{sq}-H_{sr})}{(H_p-H_r)(H_q-H_r)}\kappa(\Delta_0,\Delta_1,\Delta_2,\Delta_3,\Delta_4 \between H_p,1)(H_q,1)(H_r,1)^2 \quad \text{...(27)}.$$

Five lenses.

Here the correction to all the δ_3's may be made zero and that to $\kappa_p\delta_4$ is then

$$\frac{(H_{pq}-H_{pr})(H_{ps}-H_{pt})}{(H_q-H_r)(H_s-H_t)}\kappa(\Delta_0,\Delta_1,\Delta_2,\Delta_3,\Delta_4 \between H_q,1)(H_r,1)(H_s,1)(H_t,1) \quad \text{...(28)}.$$

Numerous fields suggest themselves as affording suitable problems for treatment by means of the foregoing equations. It is proposed to discuss a few of these in further papers.

DISCUSSION

Mr J. W. Perry: The new form for the expressions of first order aberrations here introduced by Mr Smith would appear to lend itself very readily to facile application to special problems, considerable attendant advantages in manipulation resulting also from the general symmetry by reason of the attention paid to generality. Having given, evolved already in its essentials, an optical system of whatever complexity, first steps toward the final realization of the projected system consist in gaining a knowledge as to the existence and nature of practicable solutions; this is an operation which it is highly desirable to be able to perform readily, and the methods implicitly formulated in this paper seem well adapted to this purpose.

REVIEW

Ophthalmo-Optical Manual. By WILLIAM SWAINE, B.Sc. Pp. 152; 22 Figs. (London: The Hatton Press, Ltd.) Price 5*s*. net.

According to the Preface this excellent little book is intended to be elementary, practical, and non-mathematical. It possesses all three qualities; the essential facts are described in clear and simple language, the experiments relied on are easily performed by the aid of readily accessible apparatus, and there is hardly an algebraic expression or equation in the whole book, their places being taken by numerous examples having numerals instead of letters.

The drawings are all new and appear to have been made by the author; although small they are clear and tell a great deal. The 25 tables at the end of the book contain much useful information about the eye and about spectacle lenses; many of these tables are new and the figures have obviously been calculated by the author; I have checked samples from these tables and found them correct.

Of the fifteen chapters, I and II relate to the structure of the eye and to its functions and include a clear statement of what is meant by visual acuity, III to VI are concerned with the action of prisms and lenses, VII to XII deal with the various optical errors of the eye, XIII and XIV are on retinoscopy, ophthalmoscopy, and the preliminary examination of the eye, and the last chapter gives a concise description of the various frames, mounts, and elements of spectacles.

Plane and spherical wave-surfaces are made use of in explaining the action of prisms and lenses, but the book is too elementary to extend the conception of the wave-surface to lenses when the aperture is considerable and when there is aberration. Specially clear accounts are given of the method of measuring the power of lenses by neutralization, of the effect of varying the position of the spectacle lens with respect to the eye, and of the prismatic effects due to decentering the lens.

The definition of a toric surface on p. 49 is not quite the thing. Mr Swaine says "A toriodal curve is one which has two principal curvatures in directions at right angles. Thus an arc of a circle rotated about a chord generates such a surface." The first of these statements is too wide, as every continuous curved surface has at every point on it two principal curvatures at right angles, and the second statement is too narrow, as the axis of rotation need not be a chord but may be any straight line in the plane of the circle.

The paragraph on pp. 49–50 relating to "Simple Transposition of Torics" is rather too skimpy, probably due to the non-mathematical treatment. A lens having a power of $+7D$ in the vertical meridian and $+5D$ in the horizontal meridian is to be transposed to a toric lens with a $-6D$ base curve. Nothing is said as to whether the $-6D$ curve is to be in the vertical or the horizontal meridian. There are four lenses which may be prescribed, viz.:

$$\frac{+13\text{ sph.}}{-6\text{ cyl. ax. H.} \overset{\frown}{\smile} -8\text{ cyl. ax. V.}} \;\ldots\ldots(1), \qquad \frac{+11\text{ sph.}}{-4\text{ cyl. ax. H.} \overset{\frown}{\smile} -6\text{ cyl. ax. V.}} \;\ldots\ldots(2),$$

and (3) and (4) the same as (1) and (2), but with the spherical surface last. Mr Swaine gives prescription (1) but not (2). Similarly, when the same lens is transposed to a toric lens on a $+3D$ base, the prescriptions are:

$$\frac{+4\text{ sph.}}{+3\text{ cyl. ax. H.} \overset{\frown}{\smile} +1\text{ cyl. ax. V.}} \;\ldots\ldots(5), \qquad \frac{+5\text{ cyl. ax. H.} \overset{\frown}{\smile} +3\text{ cyl. ax. V.}}{+2\text{ sph.}} \;\ldots\ldots(6),$$

and (7) and (8) the same as (5) and (6) with the surfaces reversed. Mr Swaine gives (6) but not (5). There is an obvious misprint in the last prescription of the paragraph referred to; the axis of the $11D$ cylinder should of course be vertical.

On p. 55 Mr Swaine says "In...transposing a toric or sphero-cylindrical lens, a compromise should be effected which gives an approximation towards a best form bending." If this mean that the *section* of a sphero-toric lens in either meridian should be made to approximate to the section of a best form spherical lens, that is, one designed to be free from oblique astigmatism, the statement is rather too loose as it by no means follows that such a toric lens would have the astigmatism of an oblique pencil the same as that of an axial pencil.

The book contains a great deal of information particularly useful to sight-testers and it can be confidently recommended both to students and to practising refractionists.

A. W.

ABSTRACTS OF PATENT SPECIFICATIONS

211034. Graves, A. OPTICAL PROJECTION APPARATUS.

An episcopic apparatus includes a source of light placed at the focus of an ellipsoidal reflector, and backed by a hemispherical mirror. The main reflector may be fitted with screw devices to deform its curvature so as to produce a beam of the required cross-section.

211147. May, H. OPTICAL SYSTEMS.

Behind an objective there is placed a compound prism, which splits up the light into four divergent pencils, each of which passes through an appropriate colour filter. For three-colour work, the fourth pencil is stopped off, or passed through a neutral grey filter. A second prism, having its faces parallel to those of the first prism to which they are respectively opposite, places the four images in juxtaposition upon a plate or screen. The system is reversible.

211168. Perrin, J. B., and another. TELESCOPES.

Between the eyepiece and the objective a system of prisms or mirrors is arranged in such a way that the objective can be turned in any direction while the eyepiece remains fixed. The beam is transmitted along the axis of rotation at each stage.

211476. Marcelin, A. J. COURSE INDICATORS.

An apparatus for use on the ground for following and indicating the course of an aircraft consists of a telescope system, rotatable about two axes, or including rotatable prisms, the movements executed in following the object being transmitted to a marker, which traces out upon a concave spherical surface the course followed.

211757. Berry, P. W. COLOUR SCREENS.

A single glass unit has upon it a series of merging colours, repeated several times, so as to give a number of cycles in succession without returning it to its original position, when it is slowly traversed across the lens of a projection system.

R. J. T.

Note. With regard to Mr E. Wilfred Taylor's reply to the discussion on his paper (Vol. xxv, No. 4, p. 208), Major Henrici wishes to say that he has not been connected with the Ordnance Survey since 1914 and that the opinion expressed is his own.—ED.

PROCEEDINGS
AT THE
MEETINGS OF THE OPTICAL SOCIETY
1923–24

11th October, 1923

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., Vice-President, in the Chair.

It was announced that Professor J. P. C. Southall and Mr F. E. Wright had been elected Fellows of the Society by the Council.

The following papers were read:

"Notes on the Elementary Algebraic Theory of a Class of Photographic Objectives." By E. T. HANSON, B.A.

"A General Survey of the Thin Double Lens." By T. SMITH, M.A., F.Inst.P.

"New Types of Levelling Instruments using Reversible Bubbles." By T. F. CONNOLLY, M.Sc.

27th November, 1923

Meeting held at the Imperial College of Science.

Emeritus Professor ARCHIBALD BARR, F.R.S., President, in the Chair.

The death of Dr Alexander Gleichen was announced.

The Thomas Young Oration was delivered by Professor M. von Rohr on the subject "Contributions to the History of the Spectacle Trade from the Earliest Times to Thomas Young's Appearance."

Professor M. von Rohr presented to the Society ten copies of his translation into German of the two papers by Thomas Young "On the Mechanism of the Eye" and "On Vision."

13th December, 1923

Meeting held at the Imperial College of Science.

Sir FRANK DYSON, F.R.S., Vice-President, in the Chair.

Messrs T. F. Connolly and E. F. Fincham were elected Hon. Auditors for the Society's accounts for the year 1923.

The following papers were read:

"The Primary and Secondary Constant Magnification Surfaces of Thin Lenses." By T. SMITH, M.A., F.Inst.P.

"A Suggested Standard Trial Case and Simplification in Ophthalmic Policy." By W. SWAINE, B.Sc. (Read by Mr H. H. EMSLEY).

Mr B. K. JOHNSON exhibited an Optical Revolution-Counter

2nd and 3rd January, 1924

Fourteenth Annual Exhibition of Electrical, Optical, and other Physical Apparatus held at the Imperial College of Science, jointly with the Physical Society of London.

The following discourses were given:

"The Heape and Grylls Rapid Cinema Machine." By Mr H. B. Grylls.

"The Nature and Artificial Production of Human Speech (Vowel Sounds)." By Sir Richard Paget, Bart.

10th January, 1924

Meeting held at the Imperial College of Science.

Mr R. S. Whipple, M.I.E.E., Vice-President, in the Chair.

The following papers were read:

"The Relation between Aperture, Axial Thickness, and Form for a Single Lens." By T. Smith, M.A., F.Inst.P.

"The Determination of Aberrations, as expressed in Geometrical Optics, from the Indications of the Hilger Interferometer." By J. W. Perry.

Mr David Baxandall, A.R.C.S., F.R.A.S., described Replicas of Two Galileo Telescopes, which were exhibited by the kind permission of Col. H. G. Lyons, F.R.S., Director of the Science Museum.

14th February, 1924

Annual General Meeting held at the Imperial College of Science.

Mr T. Smith, M.A., Vice-President, in the Chair.

The Hon. Secretary presented the Annual Report* of the Council, which was approved.

The Treasurer's Report† and Balance Sheet‡ were presented by the Hon. Treasurer, and were approved.

The Chairman read the list of persons nominated by the Council for office in the year 1924–25 and declared all these duly elected as no other nominations had been received.

President: Emeritus Prof. Archibald Barr.
Vice-Presidents: Sir Frank Dyson, Mr T. Smith, Instr.-Comdr. T. Y. Baker.
Business Secretary: Prof. Alan Pollard.
Papers Secretary: Mr F. F. S. Bryson.
Treasurer: Major E. O. Henrici.
Librarian: Mr J. H. Sutcliffe.
Editor: Dr John S. Anderson.

Council:

Already serving:		Elected, February, 1924:
Dr John S. Anderson	Mr P. F. Everitt.	Mr W. B. Coutts.
Mr W. M. Brett.	Dr J. W. French.	Mr A. H. Emerson.
Prof. F. J. Cheshire.	Miss L. M. Gillman.	Mr J. Guild.
Mr R. W. Cheshire.	Dr L. C. Martin.	Instr.-Lt.-Comdr. N. Langlands
Mr H. H. Emsley.	Mr A. Whitwell.	Mr F. C. Watts.

Votes of thanks to the retiring Officers and Members of Council, to the Hon. Auditors, and to the Governors of the Imperial College, with Prof. Callendar, for the use of the College for the meetings of the Society were unanimously passed.

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* See p. ix. † See p. xi. ‡ See p. xii.

14th February, 1924

Meeting held at the Imperial College of Science.

Mr T. **Smith**, M.A., Vice-President, in the Chair.

The following papers were read:

"The Addition of Aberrations." By T. **Smith**, M.A., F.Inst.P.

"The Choice of Wave-lengths for Achromatism in Telescopes." By Lt.-Col. J. W. **Gifford**, F.R.A.S.

"A New Form of Corneal Microscope with Combined Slit Lamp Illuminating Device." By E. F. **Fincham**.

Mr **David Baxandall**, A.R.C.S., F.R.A.S., described the Circular Dividing Engine of Edward Troughton, 1793, which was exhibited by the kind permission of Messrs Cooke, Troughton & Simms, Ltd., and of Col. H. G. Lyons, F.R.S., Director of the Science Museum.

13th March, 1924

Meeting held at the Imperial College of Science.

Professor F. J. **Cheshire**, C.B.E., in the Chair.

The following papers were read:

"Study of the Significance of the Foucault Knife-edge Test when applied to Refracting Systems." By Miss H. G. **Conrady**, A.R.C.S.

"The Measurement of Chromatic Aberration on the Hilger Lens Testing Interferometer." By L. C. **Martin**, D.Sc., A.R.C.S., F.R.A.S., and R. **Kingslake**.

"A Reflecting Spherometer." By B. K. **Johnson**.

"Note on a Convenient Bench for Testing Telescope Object Glasses." By L. C. **Martin**, D.Sc., A.R.C.S., F.R.A.S.

It was announced that the Council of the Physical Society of London had extended to all Fellows and Members of the Optical Society a hearty invitation to the Jubilee Celebrations to be held on the 20th, 21st, and 22nd March.

Experimental demonstrations and exhibits of apparatus were given in the laboratories of the Optical Engineering Department of the Imperial College of Science.

10th April, 1924

Meeting held at the Imperial College of Science.

Mr T. **Smith**, M.A., Vice-President, in the Chair.

The death of Mr James P. Petherick was announced.

The following papers were read:

"The Feasibility of Cinema Projection from a Continuously Moving Film." By H. **Dennis Taylor**.

"A New Perfectly Anallatic Internal Focussing Telescope." By E. **Wilfred Taylor** (Read by Mr T. **Smith**).

8th May, 1924

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., Vice-President, in the Chair.

The following papers were read:

"The Preparation of Coppered Glass Mirrors." By E. A. H. FRENCH.

"The Taylor-Hobson *F*/2 Anastigmat." By H. W. LEE, B.A. (Read by Mr T. SMITH).

"A Reference System for Primary Aberrations." By T. SMITH, M.A., F.Inst.P.

12th June, 1924

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., Vice-President, in the Chair.

The following papers were read:

"A Wide-angle Stereoscope and a Wide-angle View-finder." By Col. L. E. W. van ALBADA.

"Binocular Vision and the Stereoscopic Sense." By R. J. TRUMP, B.A., B.Sc.

"A General Solution of the First Order Aberrational Equations." By T. SMITH, M.A., F.Inst.P.

THE OPTICAL SOCIETY

REPORT OF THE COUNCIL TO THE ANNUAL MEETING OF MEMBERS, 14TH FEBRUARY, 1924

THE COUNCIL of the Optical Society presents the following Report for the year ending 31st December, 1923:

Nine Ordinary Meetings of the Society took place during the year, which were held at the Imperial College of Science and Technology by permission of the Governors and Professor Callendar, to whom the thanks and appreciation of the Society are due.

The meeting of 8th February was distinguished by the Presidential Address delivered by Sir Frank Dyson, M.A., D.Sc., LL.D., F.R.S., on "Large Telescopes," which was received by a considerable attendance of members with great interest.

At the meeting on 27th November Professor M. von Rohr delivered the Thomas Young Oration on the subject, "Contributions to the History of the Spectacle Trade from the Earliest Times to Thomas Young's Appearance." In honour of the occasion Professor von Rohr presented to the Society copies of his translation into German of the two papers by Thomas Young "On the Mechanism of the Eye," and "On Vision," for which Professor von Rohr received the thanks of the Society.

On 26th April members paid a visit to the factory of The Hadley Company at Surbiton with much interest and appreciation.

By invitation from the Astronomer Royal a visit was paid to the Royal Observatory, Greenwich, on 9th June and the thanks of the Society are due to Sir Frank Dyson and his staff for their hospitality and interesting demonstrations.

In conjunction with the Physical Society of London, the Thirteenth Annual Exhibition of Electrical, Optical and other Physical Apparatus was held at the Imperial College of Science and Technology on 3rd and 4th January, and as in previous years was largely attended.

During the year members were invited by the Board of the Institute of Physics to the series of lectures on "Physics in Industry" which have been introduced with such success, and the thanks of the Society are due to the Board for their courtesy.

The Illuminating Engineering Society invited representatives of the Optical Society to take part in a discussion on the "Projection of Light" which was held at the Royal Society of Arts on 20th February. Instructor-Commander T. Y. Baker, Mr J. Guild, and Dr L. C. Martin represented the Society on this occasion.

In the early part of the year the Society was invited by the Conjoint Board of Scientific Societies to be represented on the General Council of Honour of the International Air Congress which was held in London during June and, on the invitation of the Council, Mr A. C. W. Aldis represented the Society in this capacity.

Mr R. S. Whipple has continued to represent the Society on the National Committee for Physics and on the Conjoint Board of Scientific Societies up to its dissolution in April, while Mr J. Guild has continued to represent the Society on the Board of the Institute of Physics.

Of the special series of meetings originally initiated for the purpose of bringing into wider recognition the valuable and unique collections of optical apparatus at the South Kensington Museum, the fourth was held on 25th January when Mr Wilfred E. L. Day gave a lecture on "The Birth of Cinematography and its Antecedents." The lecture was illustrated by a large number of "magic" lantern slides and apparatus of historic interest, together with a number of historical books, amongst which were several lent by Mr T. Court. The fifth lecture of the series was delivered on 22nd March by Dr L. C. Martin on "Surveying and Navigational Instruments from the Historical Standpoint." The sixth lecture was delivered on 24th May, when Mr David Baxandall discoursed on "Early Telescopes in the Science Museum from an Historical Standpoint." Both these lectures were illustrated by many historical instruments taken from the Museum. The thanks of the Society are due to Colonel H. G. Lyons, F.R.S., Director of the Science Museum, for the loan of valuable instruments which illustrated these lectures.

It has been decided that in future Ordinary Meetings of the Society may be held occasionally at 5 p.m. instead of 7.30 p.m. when such meetings are devoted to papers of a purely scientific or mathematical nature not likely to be of direct interest to trade members.

LIBRARY

The following books have been received during 1923:

British Scientific Instrument Research Association. Fourth Annual Report for the year 1921–22.

Sprechsaal-Kalendar für die Keramischen Glas- und verwandten Industrien, 1923.

Generalised Linear Perspective. By J. W. GORDON.

Advanced Practical Physics for Students. By B. L. WORSNOP, B.Sc., and H. T. FLINT, M.Sc., Ph.D.

Mirrors, Prisms and Lenses. A textbook of Geometrical Optics. By JAMES P. C. SOUTHALL.

Photography as a Scientific Implement. (Applied Physics Series.)

Les Applications des Interférences Lumineuses. By CH. FABRY.

La Lumière Monochromatique, sa Production et son Emploi en Optique Pratique. By CH. FABRY.

Penrose's Annual: The Process Year Book, Vol. 26. Edited by WILLIAM GAMBLE, F.R.P.S., F.O.S.

No additions have been made to the journals now regularly received, the list of which is given in the Report of the Council for the year ending 31st December, 1922.

Members possessing valuable Optical Works are invited to follow the example of others who have submitted lists of their private libraries for consultation in the Library at 10, Clifford's Inn, Fleet Street, E.C.4, and have expressed their willingness to loan out any work upon request by the Librarian, Mr J. H. SUTCLIFFE. The Library will take full responsibility for any work while on loan.

MEMBERSHIP

During the year the Fellowship was conferred upon six members. Sixteen new Members were elected, of whom five are Corporate Members. Three names were removed from the list under Rule IX, 18 resignations were tendered, and two members died. There is, therefore, a decrease of seven in the membership of the Society, leaving the number of members on 1st January, 1924, as follows:

Honorary Fellows	10
Fellows	101
Fellows non-resident in British Isles	9
Members	148
Corporate Members	8
Members non-resident in British Isles	24
Corporate Member non-resident in British Isles	1
Student Members	2
TOTAL	303

TREASURER'S REPORT FOR THE YEAR 1923

THE receipts for the year are within £2 of those of last year, but as they include £42 on account of Life Compositions there is a falling off in the true income of about £40, almost entirely due to smaller receipts from sales of publications. This falling off is to some extent due to the fact that it was only possible to issue Nos. 4 and 5 of Volume XXIV of the *Transactions* just before Christmas, so that there have been no sales of these numbers apart from subscribers to the volume.

The expenditure is about £174 less than last year, the principal reduction being one of £110 in the cost of printing and distributing the *Transactions*. The net result is a balance of income over expenditure of £177. 10*s*. 4½*d*. The Society is thus in a healthy financial position.

The books and manuscripts of the Society are insured against fire with the Guardian Assurance Co., Ltd., for the sum of £300, but no credit has been taken for their value in the Balance Sheet, or for subscriptions in arrears.

E. O. HENRICI,
Hon. Treasurer.

5th January, 1924.

INCOME AND EXPENDITURE ACCOUNT FOR YEAR ENDING 31ST DECEMBER, 1923

Dr **EXPENDITURE**

	£	s.	d.
To Printing and stationery	543	11	1
Clerical assistance	102	14	3
Postage	34	9	4½
Sundries	23	11	9
Advertising	5	7	6
Physical Society—share of exhibition expenses	9	11	5
Thomas Young Oration	5	5	0
Life composition account	42	0	0
Balance—income over expenditure	177	10	4½
	£944	0	9

INCOME *Cr*

	£	s.	d.	£	s.	d.
By Subscriptions:						
Fellows, arrears	22	1	0			
,, for 1923	301	7	0			
Members, arrears	43	1	0			
,, for 1923	295	1	0			
Corporate members, arrears	3	3	0			
,, ,, for 1923	26	14	0			
Students for 1923	4	14	6			
Entrance fees	14	14	0			
	710	15	6			
Less rebate to members of the Institute of Physics	13	4	4			
				697	11	2
Life compositions				42	0	0
Sales of publications				149	0	8
Advertisements in *Transactions*, less expenses				38	10	1
Interest on investments and deposit				16	18	10
				£944	0	9

BALANCE SHEET AT 31ST DECEMBER, 1923

LIABILITIES

	£	s.	d.	£	s.	d.
To Sundry Creditors:						
Printing	223	0	9			
Advertisements	13	1	0			
Clerical assistance	13	2	6			
				249	4	3
Accumulated Funds:						
Balance 31st Dec. 1922	296	11	2			
Plus income over expenditure	177	10	4½			
				474	1	6½
Life composition fund				42	0	0
				£765	5	9½

ASSETS

	£	s.	d.
By Sundry Debtors (advertisements)	57	4	0
Cash at Bank (current account)	102	2	9
,, (deposit account)	200	0	0
Petty cash in hand	5	5	9½
Investments at cost*:			
£100 Grand Trunk Western Railway 4 % 1st Mortgage Gold Bonds	98	10	0
£300 5 % War Stock 1929/47	302	3	3
	£765	5	9½

* *Mem.* Value of Investments, market price, 31st Dec. 1923 is £378. 10*s*. 0*d*.

We, the undersigned Auditors elected at the General Meeting held on 13th December, 1923, do hereby certify that we have examined the above accounts and do find them correct.

T. F. Connolly.
E. F. Fincham.
Hon. Auditors.

E. O. HENRICI,
Hon. Treasurer.

19*th January*, 1924.